Focusing inference in defeasible argumentation

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Abstract

An argumentative system constitutes a formalization of the process of defeasible reasoning. An argument is a tentative piece of reasoning that an agent would be inclined to accept, all things considered, as an explanation for a certain hypothesis. If new information becomes available, arguments may lose support or become weakened, and no longer be regarded as valid.

In [11, 3] an argument-based reasoning system was introduced. One of the most significant aspects of this system consists in its conceptual simplicity with respect to other alternative frameworks. As a result, the inference mechanism is natural to understand, and implementation issues can be solved in a rather straightforward way.

Nevertheless, the acceptance of an argument as a valid inference involves a justification procedure, which turns out to be computationally expensive, since arguments are generated by unguided search. This represents both a simplified model of the argumentation process and higher demands on computational resources.

This paper presents some ideas about how to focus the inference process on those arguments which turn out to be decisive for the final outcome of the justification process. Preference criteria are introduced, based on the need of preserving consistency along the generation of the arguments involved.

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1 Introduction and motivations

An argumentative system [11, 6, 5, 12] constitutes a formalization of the process of defeasible reasoning. An argument A for a hypothesis h is a tentative piece of reasoning an agent would be inclined to accept, all things considered, as an explanation for h. If the agent gets new information, that argument may lose support or become weakened, so that A may no longer be regarded as valid. In that manner nonmonotonicity arises.

In *A Mathematical Treatment of Defeasible Reasoning* [11], or *MTDR*, a clear and theoretically sound structure for defining an argument-based reasoning system was introduced. Further developments in this direction were recently presented in [3]. One of the most significant aspects of the *MTDR* framework consists in its conceptual simplicity with respect to other alternative formalisms. As a result, the inference mechanism is natural and easy to understand, and implementation issues can be solved in a rather straightforward way.

Nevertheless, even though arguments are relatively easy to construct using backward chaining (they are just a special kind of proof trees), their acceptance as valid defeasible inferences involve a *justification* procedure, which turns out to be computationally expensive. In order to improve the perfomance of this procedure, some extensions have been proposed [5], which involve the use of an *arguments base*, to keep account of those arguments already generated by the system.

However, the intrinsic complexity of the specificity checking used for deciding between conflicting arguments, as well as the need of an exhaustive analysis of the search space associated with a justification are important aspects, which deserve also a special treatment. In order to obtain a justification, the existing implementations of the framework generate arguments through a trial-and-error search. This is clearly not the way we humans perform argumentation in the course of a debate, since we usually try to keep arguments "to the point", in order to support (or refute) a particular fact.

On the other hand, it is clear that in this case unguided search represents not only a simplified model of the argumentation process, but also higher demands on computational resources. This becomes even worse if we consider that specificity checking as a criterion for preferring one argument from another is quite expensive, and it must be carried out every time two conflicting arguments (argument and counterargument) are generated.

This paper presents some ideas about how to face the problems described above. First, we will show that the dialectical tree associated with a justification can be characterized as a kind of AND-OR search tree. This will help to understand why an ordering in argument generation plays a meaningful role in the perfomance of the system. Then, we will introduce a selection criterion for generating arguments as the justification process is being carried out. This selection criterion is based on a *dynamical stratification of defeasible rules*, which originates on the need of preserving consistency within argumentation lines in the debate.

2 Defeasible Argumentation

We will briefly introduce the main concepts of our framework for defeasible argumentation [11, 3] (see the appendix for definitions and further details). The knowledge of an intelligent agent $\mathcal{A}$ will be represented using a first-order language $\mathcal{L}$, plus a binary meta-linguistic relation "$\succ\!\!-$" between sets of non-ground literals of $\mathcal{L}$ which share variables. The members of this meta-linguistic relation will be called *defeasible rules*, and they have the form "$\alpha \succ\!\!- \beta$". The relation "$\succ\!\!-$" is understood as expressing that "reasons to believe in the antecedent α provide reasons to believe in the consequent β".

The beliefs of $\mathcal{A}$ are represented by a pair $(\mathcal{K}, \Delta)$, called *Defeasible Logic Structure*, where Δ is a finite set of defeasible rules. $\mathcal{K}$ represents the non-defeasible part of $\mathcal{A}$'s knowledge and Δ represents information that $\mathcal{A}$ is prepared to take at less than face value. $\Delta^{\downarrow}$ denotes the set of all ground instances of members of Δ.

An *argument* A for a conclusion h (see definition 6.4) is a set of ground defeasible rules, that together with $\mathcal{K}$ allow us to infer h. An argument constitutes a defeasible support for a conclusion h, since there may exist *better* counterarguments which *defeat* A, so that h may no longer be regarded as valid.

We will accept an argument A as a defeasible reason for a conclusion h if A is a *justification* for h. The justification process involves the construction of an acceptable argument for h. To decide the acceptability of an argument A for a literal h, its associated counterarguments (see definition 6.6) B_1, B_2, $\ldots B_k$ will be obtained, each of them being a (defeasible) reason for rejecting A. If some B_i is supported on "better" (or unrelated) evidence than A, then B_i will be a candidate for defeating A (see definition 6.8). Specificity is the preference criterion for deciding between two conflicting arguments. When specificity cannot decide, a blocking situation occurs.

Since counterarguments are also arguments, the former analysis should be in turn carried out on them. Now, B_i will defeat A unless there exists an argument C_j (which corresponds to one of the counterarguments C_1,C_2,$\ldots C_r$ associated to B_i) that defeats B_i. In that case, we will be forced to reject B_i, and hence our original argument A would be *reinstated*. If it turns out that there is a defeater D_k among the counterarguments of C_j, then B_i would be reinstated as defeater for A. Thus, the acceptance of an argument A will result from a recursive procedure, in which arguments, counterarguments, counter-counterarguments, and so on, should be taken into account. The above description leads in a natural way to the use of trees to organize that dialectical analysis.

In order to accept an argument A for a conclusion h, a *dialectical tree* can be generated (see definition 6.9). Every inner node in this tree will represent a defeater (proper or blocking), and the root of the tree will correspond to the original argument A. Nodes in this tree can in turn be recursively labeled as defeated (D-node) or undefeated nodes (U-node). If all children nodes of the root turn out to be labeled as defeated, we say that A is an *acceptable argument*. The procedure just described closely resembles a logical discussion or argumentation, *i.e.*, it is a *dialectical* process. This characterization leads us to a labeling procedure, after which we can conclude whether the root of the dialectical tree corresponds indeed to an acceptable argument.

Besides, as arguments are labeled as U-nodes or D-nodes within a dialectical tree, they are stored in an arguments base [5]. In that way, arguments appearing several times in the same dialectical tree are generated and labeled just once. Additional occurences of those arguments and their associated labels can be retrieved from the arguments base, without having to construct them again.

3 Speeding up inference

As we have shown in [3], the number of nodes (arguments) in a dialectical tree is finite. Nevertheless, this number could be large enough for making exhaustive search impossible within reasonable time constraints. In this section, we will introduce a pruning strategy which allows us to reduce the search space associated with the dialectical argumentation process.

According to the definition 6.10 (see appendix), nodes (arguments) in a dialectical tree can be recursively label as *U-nodes* (undefeated arguments) or *D-nodes* (defeated arguments). In order to label a node as "undefeated", all his child nodes (defeaters) must be defeated. Similarly, a node can be labeled as "defeated" if it has at least one undefeated child node (defeater). That way, as far as the labeling procedure is concerned, a dialectical tree is a kind of AND-OR tree: U-nodes correspond to AND-nodes, and D-nodes to OR-nodes.

This feature allows us to carry out a pruning strategy on a dialectical tree (similar to α-β pruning [4]). Thus, in order to determine if a given argument is or not a justification, it will not be necessary to perform an exhaustive analysis involving every argument in the dialectical tree. As a result, the inference process can be speeded up.

EXAMPLE **3.1** Consider the following dialectical tree. The arguments marked with an asterisk had not to be taken into consideration in order to label the root of the tree as D (defeated).

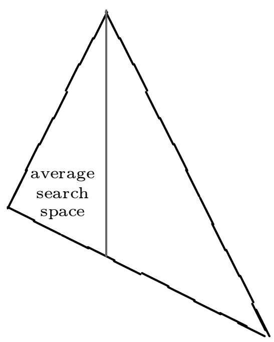
```
         D
       / | \
      /  |  \
     /   |   \
    /    |    \
   /     |     \
  U      *      *
 / \    / \     |\
/   \  *   *    | \
D   D           *  *
   /| \        / \
  / |  \      /   \
 U  *   *    *     *
```

□

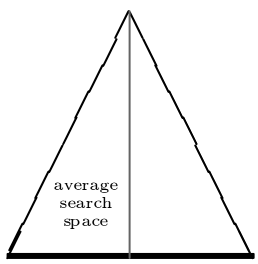
The possibility of performing such a pruning on the dialectical tree shows clearly that the *order* in which arguments (counterarguments, counter-counterarguments, etc.) are to be considered plays a meaningful role. Not every argumentation line[1] in the dialectical tree with root A must be tested in order to determine if the argument A is indeed a justification.

Since most arguments do not have the same number of counterarguments, a dialectical tree will be seldom balanced. The depth of the tree will correspond to the longest argumentation line in the tree. According to our preceding analysis, the evaluation for labeling the arguments on this argumentation line should be delayed as long as possible, since –all things being equal– shorter argumentation lines have the same chances of "breaking the debate", and they can be evaluated faster than longer ones. This fact implies that the size of the search space associated with the justification procedure will be different, depending on the order in which argumentation lines are going to be generated. Thus, if the first counterargument taken into consideration turns out to remain undefeated, the size of the search space will be minimal. The worst case will arise when we need to analize all the search space.

Assuming that argumentation lines within a dialectical tree were sorted on their length (this can be done recursively), a *sorted* dialectical tree would result, as shown on the left in the figure below.



A sorted dialectical tree



An average dialectical tree

[1]An argumentation line in a dialectical tree is a path from the root to a leaf (see definition 6.12)

Given an arbitrary dialectical tree, we analize on the average half of the tree (assuming depth-first search), and this should be on the average half of the search space (*i.e.*, we should consider half of the number of arguments in that dialectical tree). As we can see from the drawing above, half of the sorted tree is clearly less than half of the whole search space, so that the pruning strategy would have its best perfomance if it were carried out on such a tree.

Thus, in order to prune the search space, it turns out to be particulary important to have a criterion for preferring the *most promising* counterargument first. The words "most promising" mean here "that one belonging to the shortest argumentation line". Should this counterargument be defeated, then the next argumentation line must be again kept as short as possible, and so on. The longest argumentation line should be delayed as long as possible, since on the average, it will be the one which involves the most complex analysis.

However, there are several intuitive reasons which lead us to believe that such a criterion could not be applied in defeasible argumentation. As Vreeswijk [12] correctly observes, the only way of actually determining the strenght of an argument is throwing it into the debate. We cannot know beforehand how many counterarguments a particular argument could have. A counterargument which looks strong enough for defeating a given argument may have more counter-counterarguments than a weaker counterargument without counter-counterarguments. In this respect, Vreeswijk ([12], page 151) observes that:

> ... But would it not be a good idea, for example, to let each party present its strongest arguments first? In that way, the discussion is not distracted by weak arguments that hold up the progress, so that interests are soon heading towards the right direction. Or, if that idea does not work, then maybe it will help to forbid each party to play out the less promising arguments first, again to ensure that needless detours in the dialectical structure will be avoided. Unfortunately, both suggestions do not really contribute to a solution of the problem. The last suggestion is even useless, since there is no other way of finding out whether an argument is promising or not, then to actually 'throw' it into the discussion. In fact, it begs the whole question of what is involved in testing the credibility of an argument. So this suggestion does not help us much further. The first suggestion, although it is valid, is not of much use either, since the conclusive force of an argument does not say much about its effectivity in the course of the debate. ...

We claim that some criteria can be given, based on some theoretical considerations, in order to know in advance how promising an argument can be in the course of a debate. These criteria will be formally stated and discussed in the next section.

4 Stratifying defeasible rules

What determines when a counterargument is more promising than other for breaking a debate? This question is not an easy one. Appealing to our intuition, "good" counterarguments are those which are "difficult" to refute. As a first approach, we can then say that, in general, argumentation lines will be shorter when *every* argument involved has as few literals as possible at which it can be further counterargued. Thus, the branching factor of the dialectical tree would be minimized.

As already shown in [**?**], our framework introduces an important constraint for constructing argumentation lines: supporting and interferring arguments must be consistent among themselves. Thus, an interferring (supporting) argument that counterargues a supporting (interferring) argument is related to previous interferring (supporting) arguments within the same argumentation line.

In other words: *once a supporting (interferring) argument A has been thrown into the debate, any other supporting (interferring) argument in further stages of the debate is forced not to contradict those conclusions that can be inferred from A. Breaking this rule causes supporting (interferring) arguments to no longer be consistent among themselves.*

This leads us to the following claim:

CLAIM **4.1** Let $\langle A, h\rangle$ be an argument structure, and let $\langle B, q\rangle$ be a counterargument for $\langle A, h\rangle$. On the average, $\langle B, q\rangle$ will be a 'good' counterargument if $A \cap B$ is as large as possible. $\Box$

That is to say: a S-argument (I-argument) which uses as much information as possible from previous I-arguments (S-arguments) will have, on the average, better chances of remaining undefeated than ordinary counterarguments obtained randomly from the knowledge base.

Besides, the structural resemblance of argument and counterargument would help simplify the specificity checking, by eliminating the analysis on literals supported on the same set of grounded defeasible rules in both the argument and the counterargument involved. This structural resemblance can be established without considering the system's KB, from which arguments have been obtained. Consider the following example.

EXAMPLE **4.1** Let

$$\begin{array}{lcl} A & = & \langle \{d \succ\!\!- b, e \succ\!\!- c, b \wedge c \succ\!\!- a\}, a \rangle \\ B_1 & = & \langle \{d \succ\!\!- b, e \succ\!\!- c, b \wedge c \wedge r \succ\!\!- \neg a\}, \neg a \rangle \\ B_2 & = & \langle \{p \succ\!\!- \neg a\}, \neg a \rangle \\ B_3 & = & \langle \{g \wedge h \succ\!\!- \neg a\}, \neg a \rangle \end{array}$$

be argument structures. Then B_1,B_2 and B_3 are counterarguments for A.

Why does B_1 seems to be a 'good' counterargument at first sight? We know immediatly that it is more specific than A; we cannot assure anything about specificity for B_2 or B_3, even though they are structurally simpler than B_1. Were these arguments the only ones for concluding a and $\neg a$, we would also tend to think that B_1 defeats A, without pursing the analysis any further.

Why? If an argument C attacks B_1 at an inner literal, this would mean breaking the rule of keeping supporting argument consistent among themselves. Thus, in order to counterargue B_1, a counter-counterargument C must necessarily attack B_1's conclusion (*i.e.*, $\neg a$). The same analysis would apply to an interferring argument D, counterarguing C, with respect to the defeasible rules used in B_1; interferring arguments must be also consistent among themselves. □

We will introduce some definitions, in order to formalize our claim. First, since ground defeasible rules represent material implications within an argument structure, we have to be able to deal with them as such.

Definition 4.1 Let $\langle A, h \rangle$ be an argument structure. Then $\underline{A}$ denotes a set of material implications, where every ground defeasible rule $a \succ\!\!- b$ in A is replaced by $a \rightarrow b$ in $\underline{A}$.

This definition can be extended to handle a set of argument structures.

Definition 4.2 Let S be a set of argument structures $\{ \langle A_1, h_1 \rangle, \langle A_2, h_2 \rangle, \ldots, \langle A_n, h_n \rangle \}$. Then $\underline{S}$ denotes the set $\underline{A_1} \cup \underline{A_2} \cup \ldots \cup \underline{A_n}$.

The set of supporting and interferring arguments up to a certain point in an argumentation line can be characterized as follows.

Definition 4.3 Let $[A_0, A_1, A_2, \ldots, A_n]$ be an argumentation line. Then

$$S_\lambda{}^i = \{\ A_k \text{ such that } k = 2p,\, p \geq 0,\ k \quad i\}$$
$$I_\lambda{}^i = \{\ A_k \text{ such that } k = 2p+1,\, p \geq 0,\ k \quad i\}$$

Definition 4.4 Let $\langle A, h\rangle$ be an argument structure. Then $\mathrm{Csc}(\langle A, h\rangle)$ denotes the set of consequents of ground defeasible rules in A.

Definition 4.5 Let S be a set of argument structures $\{\ \langle A_1, h_1\rangle, \langle A_2, h_2\rangle, \ldots, \langle A_n, h_n\rangle\ \}$. Then $\mathrm{Csc}(S) = \bigcup\ \{\mathrm{Csc}(\langle A, h\rangle) : \forall \langle A, h\rangle \in S\ \}$.

The efectiveness of our former claim becomes now clear with the following lemma, whose proof is straightforward form the concepts discussed so far:

LEMMA **4.1** *Let* $\lambda = [A_0, A_1, A_2, \ldots, A_n]$ *be an argumentation line. Then the set*
$C = \underline{S_\lambda{}^i} \cap \underline{I_\lambda{}^i}$ *constitutes non-defeasible knowledge for the construction of the arguments* $[A_{i+1}, A_{i+2}, \ldots, A_n]$.

When constructing interferring (supporting) arguments, we will prefer those defeasible rules already used by previous supporting (interferring) arguments in the same argumentation line, since the conclusions of these rules *cannot* be counter-counterargued because of the consistency constraints described before.

From the previous lemma, it is also clear that

- *Ground defeasible rules* present in both a supporting and an interferring argument within the same argumentation line can be thought of as *material implications* of the system's knowledge base.
- *Literals* supported on subarguments in a supporting (interferring) argument can be thought of as *grounded facts* of the system's knowledge base, in order to construct other supporting (interferring) arguments within the same argumentation line.

As an argumentation line is being constructed, the sets $\mathcal{K}_P$ and $\mathcal{K}_G$ expand dynamically as we go deeper into the dialectical tree, and shrink as we discard arguments which turned out to be defeated. The augmented sets correspond to two different supersets: one containing the ground defeasible rules belonging to supporting arguments in an argumentation line, and the other contains those rules that belong to interferring arguments in that argumentation line. On the other hand, as stated in the lemma above, those grounded defeasible rules shared by an S-argument and an I-argument in the same argumentation line can be considered to be nondefeasible knowledge for further stages of the debate within that argumentation line.

As the debate progresses, the possibility of counterarguing becomes more difficult, since the rules used for constructing arguments in the first stages impose consistency constraints for the formulation of new arguments, and conclusions based on defeasible knowledge (accepted by both parties in the debate) are no longer questionable, being its epistemic status similar to particular facts.

Then, as λ is being constructed, the system's knowledge base would be expanded as follows (the numbers on the right column correspond to the current depth of the argumentation line).

	$\mathcal{K}_G$ for S-arguments	$\mathcal{K}_G$ for I-arguments
0	$\mathcal{K}_G$	$\mathcal{K}_G$
1	$\mathcal{K}_G \cup \underline{S_\lambda{}^1}$	$\mathcal{K}_G \cup \underline{I_\lambda{}^1}$
2	$\mathcal{K}_G \cup \underline{S_\lambda{}^2}$	$\mathcal{K}_G \cup \underline{I_\lambda{}^2}$
3	$\mathcal{K}_G \cup \underline{S_\lambda{}^3}$	$\mathcal{K}_G \cup \underline{I_\lambda{}^3}$
...		
k	$\mathcal{K}_G \cup \underline{S_\lambda{}^k}$	$\mathcal{K}_G \cup \underline{I_\lambda{}^k}$

	$\mathcal{K}_P$ for S-arguments	$\mathcal{K}_P$ for I-arguments
0	$\mathcal{K}_P$	$\mathcal{K}_P$
1	$\mathcal{K}_P \cup \mathrm{Csc}(S_\lambda{}^1)$	$\mathcal{K}_P \cup \mathrm{Csc}(I_\lambda{}^1)$
2	$\mathcal{K}_P \cup \mathrm{Csc}(S_\lambda{}^2)$	$\mathcal{K}_P \cup \mathrm{Csc}(I_\lambda{}^2)$
3	$\mathcal{K}_P \cup \mathrm{Csc}(S_\lambda{}^3)$	$\mathcal{K}_P \cup \mathrm{Csc}(I_\lambda{}^3)$
...		
k	$\mathcal{K}_P \cup \mathrm{Csc}(S_\lambda{}^k)$	$\mathcal{K}_P \cup \mathrm{Csc}(I_\lambda{}^k)$

	Augmented $\mathcal{K}_G$
0	$\mathcal{K}_G$
1	$\mathcal{K}_G \cup \{ S_\lambda^1 \cap I_\lambda^1 \}$
2	$\mathcal{K}_G \cup \{ S_\lambda^2 \cap I_\lambda^2 \}$
3	$\mathcal{K}_G \cup \{ S_\lambda^3 \cap I_\lambda^3 \}$
...	
k	$\mathcal{K}_P \cup \{ S_\lambda^k \cap I_\lambda^k \}$

The preference ordering used for constructing I-arguments (S-arguments) will be the following:

1. Use $\mathcal{K}_P$ and $\mathcal{K}_G$ for S-arguments (I-arguments), if possible; else
2. Use augmented $\mathcal{K}_G$; else
3. Use defeasible rules from Δ.

5 Conclusions

The need of preserving consistency within a debate has proven to be an important issue in argumentative reasoning. In [3], by introducing a dialectics-based approach, we were able to detect fallacious argumentation. The problem was solved by introducing some consistency constraints on argumentation lines.

In this paper, we have discussed the intuitive ideas which led us to think about inference and its relation to consistency in argumentation lines. We were able to distinguish the knowledge on which S-arguments and I-arguments are supported. This consistency-based approach seems to give us criteria for guiding the debate, in order to find out if our original argument is a justification or not. These criteria allowed us to refine the original, simplified model of the process carried out for constructing arguments (exhaustive search). As a result, the use of the computational resources needed for performing argumentation could be improved.

We think that this topic needs further research, in order to incorporate it into the existing frameworks for argumentative reasoning. A more precise formalization of the ideas discussed in this paper is being worked on at the time.

6 Appendix: a framework for defeasible argumentation

In this appendix, the main definitions of a defeasible argumentation framework are introduced (see [11] and [3] for further details). The knowledge of an intelligent agent $\mathcal{A}$ will be represented using a first-order language $\mathcal{L}$, plus a binary meta-linguistic relation "$\succ\!\!-$" between sets of non-ground literals of $\mathcal{L}$ which share variables. The members of this meta-linguistic relation will be called *defeasible rules*. The defeasible rule "$\alpha \succ\!\!- \beta$" is to be understood as expressing that "reasons to believe in the antecedent α provide reasons to believe in the consequent β".

Definition **6.1** Let $\mathcal{K}$ be a consistent subset of sentences of the language $\mathcal{L}$, called the *context*. This set can be partitioned in two subsets $\mathcal{K}_G$, of *general* (necessary) knowledge, and $\mathcal{K}_P$, of *particular* (contingent) knowledge.

Definition **6.2** The pair $(\mathcal{K}, \Delta)$, called *Defeasible Logic Structure*, represents the beliefs of $\mathcal{A}$. The set $\mathcal{K}$ corresponds to the non-defeasible part of $\mathcal{A}$'s knowledge. The set Δ is a finite set of defeasible rules, representing information that $\mathcal{A}$ is prepared to take at less than face value. $\Delta^{\downarrow}$ will denote the set of all ground instances of members of Δ.

Definition **6.3** Let Γ be a subset of $\mathcal{K} \cup \Delta^{\downarrow}$. A ground literal h is a *defeasible consequence* of Γ, abbreviated $\Gamma \mid\!\sim h$, if and only if there exists a finite sequence $B_1, \ldots, B_n$ such that $B_n = h$ and for $1 \quad i < n$, either $B_i \in \Gamma$, or B_i is a direct consequence of the preceding elements in the sequence by virtue of the application of any inference rule of the first-order theory associated with the language $\mathcal{L}$. The ground instances of the defeasible rules are regarded as material implications for the application of inference rules. We will also write $\mathcal{K} \cup A \mid\!\sim h$ distinguishing the set A of defeasible rules used in the derivation from the context $\mathcal{K}$.

Definition **6.4** We say that a subset A of $\Delta^{\downarrow}$ is an *argument structure* for h in the context $\mathcal{K}$ (denoted by $\langle A, h \rangle_{\mathcal{K}}$, or just $\langle A, h \rangle$) iff: (1) $\mathcal{K} \cup A \mid\!\sim h$, (2) $\mathcal{K} \cup A \not\mid\!\sim \bot$ and (3) $\not\exists A' \subset A$, $\mathcal{K} \cup A' \mid\!\sim h$. A *subargument* of $\langle A, h \rangle$ is an argument $\langle S, j \rangle$ such that $S \subseteq A$. Given $\langle A, h \rangle_{\mathcal{K}}$, we can also say that *$A$ is an argument for h*.

Example 6.1 Given $(\mathcal{K}, \Delta)$, $\mathcal{K} = \{d \rightarrow b, d, f, l\}$, $\Delta = \{b \wedge c \succ\!\!- h, f \succ\!\!- c, l \wedge f \succ\!\!- \neg c\}$, we say that $\langle \{f \succ\!\!- c, b \wedge c \succ\!\!- h\}, h\rangle$ is an argument structure for h. $\square$

Definition 6.5 Two argument structures $\langle A_1, h_1\rangle$ and $\langle A_2, h_2\rangle$ *disagree*, denoted $\langle A_1, h_1\rangle \bowtie \langle A_2, h_2\rangle$, if and only if $\mathcal{K} \cup \{h_1, h_2\} \vdash \bot$.

Definition 6.6 We say that $\langle A_1, h_1\rangle$ *counterargues* $\langle A_2, h_2\rangle$ at literal h, denoted $\langle A_1, h_1\rangle \overset{h}{\rightarrow} \langle A_2, h_2\rangle$ iff

1. There exists a subargument $\langle A, h\rangle$ of $\langle A_2, h_2\rangle$ such that $\langle A_1, h_1\rangle \bowtie \langle A, h\rangle$.
2. For every proper subargument $\langle S, j\rangle$ of $\langle A_1, h_1\rangle$, it is not the case that $\langle A_2, h_2\rangle \rightarrow \langle S, j\rangle$.

If $\langle A_1, h_1\rangle \overset{h}{\rightarrow} \langle A_2, h_2\rangle$, we can also say that $\langle A_1, h_1\rangle$ *is a counterargument for* $\langle A_2, h_2\rangle$.

Definition 6.7 Let $\mathcal{D} = \{a \in \mathcal{L} : a$ is a ground literal and $\mathcal{K} \cup \Delta^{\downarrow} \mid\!\sim a\}$, and let $\langle A_1, h_1\rangle$,$\langle A_2, h_2\rangle$ be two argument structures. We say that A_1 for h_1 is *strictly more specific than* A_2 for h_2, denoted $\langle A_1, h_1\rangle \succ_{\mathbf{spec}} \langle A_2, h_2\rangle$, if and only if

i) $\forall S \subseteq \mathcal{D}$ if $\mathcal{K}_G \cup S \cup A_1 \mid\!\sim h_1$ and $\mathcal{K}_G \cup S \not\mid\!\sim h_1$, then $\mathcal{K}_G \cup S \cup A_2 \mid\!\sim h_2$.
ii) $\exists S \subseteq \mathcal{D}$ such that $\mathcal{K}_G \cup S \cup A_2 \mid\!\sim h_2$, $\mathcal{K}_G \cup S \not\mid\!\sim h_2$ and $\mathcal{K}_G \cup S \cup A_1 \not\mid\!\sim h_1$.

Definition 6.8 $\langle A_1, h_1\rangle$ *defeats* $\langle A_2, h_2\rangle$ at literal h, denoted $\langle A_1, h_1\rangle \gg_{\mathbf{def}} \langle A_2, h_2\rangle$, if and only if there exists a subargument $\langle A, h\rangle$ of $\langle A_2, h_2\rangle$ such that: $\langle A_1, h_1\rangle$ counterargues $\langle A_2, h_2\rangle$ at the literal h and

1. $\langle A_1, h_1\rangle$ is strictly more specific than $\langle A, h\rangle$, or
2. $\langle A_1, h_1\rangle$ is unrelated by specificity to $\langle A, h\rangle$.

In case (1) $\langle A_1, h_1\rangle$ will be called a *proper* defeater, and in case (2) a *blocking* defeater. If $\langle A_1, h_1\rangle$ *defeats* $\langle A_2, h_2\rangle$, we will also say that $\langle A_1, h_1\rangle$ is a *defeater* for $\langle A_2, h_2\rangle$.

Example 6.2 Given $(\mathcal{K}, \Delta)$ as defined in example 6.1, we have the following relations between arguments.

$$\langle \{l \wedge f \succ\!\!- \neg c\}, \neg c\rangle \bowtie \langle \{f \succ\!\!- c\}, c\rangle$$
$$\langle \{l \wedge f \succ\!\!- \neg c\}, \neg c\rangle \overset{c}{\rightarrow} \langle \{f \succ\!\!- c, b \wedge c \succ\!\!- h\}, h\rangle$$
$$\langle \{l \wedge f \succ\!\!- \neg c\}, \neg c\rangle \succ_{\mathbf{spec}} \langle \{f \succ\!\!- c\}, c\rangle$$
$$\langle \{l \wedge f \succ\!\!- \neg c\}, \neg c\rangle \gg_{\mathbf{def}} \langle \{f \succ\!\!- c, b \wedge c \succ\!\!- h\}, h\rangle$$

□

DEFINITION **6.9** Let $\langle A, h\rangle$ be an argument structure. A *dialectical tree* for $\langle A, h\rangle$, denoted $\mathcal{T}_{\langle A,\, h\rangle}$, is recursively defined as follows:

1. A single node containing an argument structure $\langle A, h\rangle$ with no defeaters (proper or blocking) is by itself a dialectical tree for $\langle A, h\rangle$. This node is also the root of the tree.
2. Suppose that $\langle A, h\rangle$ is an argument structure with defeaters (proper or blocking) $\langle A_1, h_1\rangle$, $\langle A_2, h_2\rangle$, ..., $\langle A_n, h_n\rangle$. We construct the dialectical tree for $\langle A, h\rangle$, $\mathcal{T}_{\langle A,\, h\rangle}$, by putting $\langle A, h\rangle$ in the root node of it and by making this node the parent node of the roots of the dialectical trees of $\langle A_1, h_1\rangle$, $\langle A_2, h_2\rangle$, ..., $\langle A_n, h_n\rangle$, *i.e.*, $\mathcal{T}_{\langle A_1,\, h_1\rangle}$, $\mathcal{T}_{\langle A_2,\, h_2\rangle}$, ..., $\mathcal{T}_{\langle A_n,\, h_n\rangle}$. If an unacceptable argument line gets formed (see definition 6.15), during the construction of this tree, it suffices to clip the subtree rooted in the offending argument that violates a condition in the definition of acceptable argumentation line.

DEFINITION **6.10** Let $\mathcal{T}_{\langle A,\, h\rangle}$ be a dialectical tree for an argument structure $\langle A, h\rangle$. The nodes of $\mathcal{T}_{\langle A,\, h\rangle}$ can be recursively labeled as *undefeated nodes* (U-nodes) and *defeated nodes* (D-nodes) as follows:

1. Leaves of $\mathcal{T}_{\langle A,\, h\rangle}$ are *U-nodes*.
2. Let $\langle B, q\rangle$ be an inner node of $\mathcal{T}_{\langle A,\, h\rangle}$. Then $\langle B, q\rangle$ will be an *U-node* iff every child of $\langle B, q\rangle$ is a *D-node*. $\langle B, q\rangle$ will be a *D-node* iff it has at least an *U-node* as a child.

DEFINITION **6.11** Let $\langle A, h\rangle$ be an argument structure, and let $\mathcal{T}_{\langle A,\, h\rangle}$ be its associated dialectical tree. We will say that A is a *justification* for h (or simply $\langle A, h\rangle$ is a *justification*) iff the root node of $\mathcal{T}_{\langle A,\, h\rangle}$ is an U-node.

According to this definition, an argumentative knowledge-based system has four possible answers for a given query h.

- YES, if there is a justification $\langle A, h\rangle$.
- NO, if for every possible argument structure $\langle A, h\rangle$, there exists a justification for at least one proper defeater of $\langle A, h\rangle$.
- UNKNOWN, if there exists no argument structure $\langle A, h\rangle$.
- UNDEFINED, if for every possible argument structure $\langle A, h\rangle$, there are no proper defeaters for $\langle A, h\rangle$, but there exists at least one blocking defeater for $\langle A, h\rangle$.

Now we will introduce two additional concepts, already suggested in [11], which will prove to be useful in what follows.

Definition **6.12** Let $\langle A_0, h_0 \rangle$ be an argument structure, and let $\mathcal{T}_{\langle A_0, h_0 \rangle}$ be its associated dialectical tree. Then every path λ in $\mathcal{T}_{\langle A_0, h_0 \rangle}$ from the root $\langle A_0, h_0 \rangle$ to a leaf $\langle A_n, h_n \rangle$, denoted $\lambda = [\ \langle A_0, h_0 \rangle,\ \langle A_1, h_1 \rangle,\ \langle A_2, h_2 \rangle,\ \ldots,\ \langle A_n, h_n \rangle]$, constitutes an *argumentation line* for $\langle A_0, h_0 \rangle$.

Definition **6.13** Let $\mathcal{T}_{\langle A_0, h_0 \rangle}$ be a dialectical tree, and let $\lambda = [\ \langle A_0, h_0 \rangle,\ \langle A_1, h_1 \rangle,\ \langle A_2, h_2 \rangle,\ \ldots,\ \langle A_n, h_n \rangle]$ be an argumentation line for $\langle A_0, h_0 \rangle$. Then every $\langle A_i, h_i \rangle$ in λ can be labeled as a *supporting* or *interfering* argument as follows

1. $\langle A_0, h_0 \rangle$ is a supporting argument in λ, and
2. If $\langle A_i, h_i \rangle$ is a supporting (interfering) argument in λ, then $\langle A_{i+1}, h_{i+1} \rangle$ is an interfering (supporting) argument in λ.

We will denote as S_λ and I_λ the set of all supporting and interfering arguments in λ, respectively.

As we can see from this definition, an argumentation line λ can now be thought of as an alternate sequence of supporting and interfering arguments as in any ordered debate.

Definition **6.14** Given two argument structures $\langle A_1, h_1 \rangle$ and $\langle A_2, h_2 \rangle$ we will say that they are *concordant* iff $\mathcal{K} \cup A_1 \cup A_2 \not\vdash \perp$. In general, a family of argument structures $\{\langle A_i, h_i \rangle\}_{i=1}^{n}$ is concordant iff $\mathcal{K} \cup \bigcup_{i=1}^{n} A_i \not\vdash \perp$.

Definition **6.15** Let $\lambda = [\langle A_0, h_0 \rangle, \langle A_1, h_1 \rangle, \langle A_2, h_2 \rangle, \ldots, \langle A_n, h_n \rangle]$ be an argumentation line. Then, λ will be called an *acceptable argumentation line* iff

1. Supporting (interfering) arguments in λ are concordant pairwise, *i.e.*, $\mathcal{K} \cup A_i \cup A_j \not\vdash \perp$, for every $\langle A_i, h_i \rangle, \langle A_j, h_j \rangle \in S_\lambda$ (I_λ).
2. Let $\langle A_i, h_i \rangle$ be an argument structure in S_λ (I_λ). There is no argument $\langle A_j, h_j \rangle$ in I_λ (S_λ), such that $i < j$ and $\langle A_i, h_i \rangle$ defeats $\langle A_j, h_j \rangle$.

The first condition causes supporting (interferig) arguments in an argumentation line to be consistent among themselves. The second condition prevents circularity, forcing interfering arguments not to be defeated by previous arguments in a given argumentation line. Interfering arguments must be constructed considering which arguments have been offered already.

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