# Strong and Weak Forms of Abstract Argument Defense

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**Abstract.** Extended abstract frameworks separate conflicts and preference between arguments. These elements are combined to induce argument defeat relations. A proper defeat is consequence of preferring an argument in a conflicting pair, while blocking defeat is consequence of incomparable or equivalent-in-strength conflicting arguments. As arguments interact with different strengths, the quality of several argument extensions may be measured in a particular semantics. In this paper we analyze the strength of defenses in extended argumentation frameworks, under admissibility semantics. A more flexible form of acceptability is defined leading to a credulous position of acceptance.

**Keywords.** Abstract argumentation, admissibility semantics, credulous acceptance.

## 1. Introduction

Abstract argumentation systems [9,14,1,2] are formalisms for argumentation where some components remain unspecified towards the study of pure semantic notions. Most of the existing proposals are based on the single abstract concept of *attack* represented as a binary relation, and according to several rational rules, extensions are defined as sets of possibly accepted arguments. The attack relation is basically a subordinate relation of conflicting arguments. For two arguments $\mathcal{A}$ and $\mathcal{B}$, if $(\mathcal{A}, \mathcal{B})$ is in the attack relation, then the status of acceptance of $\mathcal{B}$ is conditioned by the status of $\mathcal{A}$, but not the other way around. It is said that argument $\mathcal{A}$ attacks $\mathcal{B}$, and it implies a priority between conflicting arguments. It is widely understood that this priority is related to some form of argument strength. This is easily modeled through a preference order. Several frameworks do include an argument order [1,3,5], although the classic attack relation is kept, and therefore this order is not used to induce attacks.

In [11,10] an extended abstract argumentation framework is introduced, where two kinds of defeat relations are present. These relations are obtained by applying a preference criterion between conflictive arguments. The conflict relation is kept in its most basic, abstract form: two arguments are in conflict simply if both arguments cannot be accepted simultaneously. The preference criterion subsumes any evaluation on arguments and it is used to determine the direction of the attack. Consider the following arguments, showing a simple argument defense:

- $Fly_1$: Fly Oceanic Airlines because it has the cheapest tickets.
- $NoFly$: Do not fly Oceanic Airlines because the accident rate is high and the onboard service is not good.
- $Fly_2$: Fly Oceanic Airlines because the accident rate is normal and the onboard service is improving.
- $Fly_3$: Fly Oceanic Airlines because you can see some islands in the flight route.

There is a conflict between $NoFly$ and the other arguments. This conflict is basically an acceptance constraint: if $NoFly$ is accepted, then $Fly_1$ should be rejected (as well as $Fly_2$ and $Fly_3$) and viceversa. A comparison criterion may be applied towards a decision for argument acceptance. Suppose the following preferences are established. Argument $NoFly$ is considered a stronger argument than $Fly_1$, as it includes information about accidents and service, which is more important than ticket fares. Then $NoFly$ is said to be a *proper defeater* of $Fly_1$. Argument $Fly_2$ is also referring to safety and service, and therefore is considered as strong as $NoFly$. Both arguments are blocking each other and thus they are said to be *blocking defeaters*. Argument $Fly_3$ is referring to in-flight sightseeing, which is not related to the topics addressed by $NoFly$. Both arguments are incomparable to each other, and they are considered blocking defeaters. This situation can be depicted using graphs, with different types of arcs. Arguments are represented as black triangles. An arrow ($\leftarrow$) is used to denote proper defeaters. A double-pointed straight arrow ($\leftrightarrow$) connects blocking defeaters considered equivalent in strength, and a double-pointed zig-zag arrow ($\leftrightsquigarrow$) connects incomparable blocking defeaters. In Figure 1, the previous arguments and its relations are shown. Argument $NoFly$ is a proper defeater of $Fly_1$. Arguments $Fly_2$ and $NoFly$ are equivalent-in-strength blocking defeaters, and arguments $Fly_3$ and $NoFly$ are incomparable blocking defeaters.

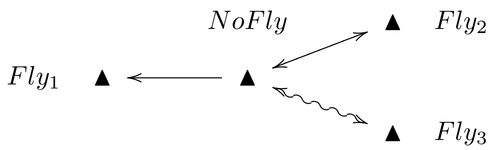


**Figure 1.** EAF-graph

In this example, arguments $Fly_2$ and $Fly_3$ are defeaters of $NoFly$, which is a defeater of $Fly_1$. Therefore, $Fly_1$ is said to be defended by $Fly_2$ and $Fly_3$. Note that the best defense should be provided by an argument considered stronger than $NoFly$. Here, defense is achieved by blocking defeaters, that is, by arguments in symmetric opposition. Even then, the defenses are of different quality. The defense provided by $Fly_2$ may be considered stronger than the one provided by $Fly_3$, as it is possible to compare $Fly_2$ to $NoFly$, although just to conclude they have equivalent strength. The argument $Fly_3$ cannot be compared to $NoFly$ and then the conflict remains unevaluated. Incomparability arise because of incomplete information, or because the arguments are actually incomparable. This may be viewed as the weakest form of defense. These different grades of defense, achieved as a result of argument comparison, is the main motivation of this work.

In this paper we formalize the strength of defenses for arguments and we explore its relation to classic semantic notions. This paper is organized as follows. In the next section, the extended argumentation frameworks are formally introduced. In Section 3 the notion of admissibility is applied to extended frameworks. In Section 4, the strength of defenses is formalized and this notion is applied to analyze the grounded extension in Section 5. In order to adopt a credulous position, a more flexible form of acceptability is defined in Section 6. Finally, the conclusions are presented in Section 7.

## 2. Extended Abstract Frameworks

In our extended argumentation framework three relations are considered: *conflict*, *subargument* and *preference* between arguments. The definition follows:

**Definition 1 (Extended Framework)** *An extended abstract argumentation framework (called EAF) is a quartet* $\Phi$=$\langle Args, \sqsubseteq, \mathbf{C}, \mathbf{R}\rangle$*, where* $Args$ *is a finite set of arguments, and* $\sqsubseteq$*,* $\mathbf{C}$ *and* $\mathbf{R}$ *are binary relations over* $Args$ *denoting respectively subarguments, conflicts and preferences between arguments.*

Arguments are abstract entities, as in [9], that will be denoted using calligraphic uppercase letters, possibly with indexes. In this work, the subargument relation is not relevant for the topic addressed. Basically, it is used to model the fact that arguments may include inner pieces of reasoning that can be considered arguments by itself, and it is of special interest in dialectical studies [12]. Hence, unless explicitly specified, in the rest of the paper $\sqsubseteq = \emptyset$ . The conflict relation $\mathbf{C}$ states the incompatibility of acceptance between arguments, and thus it is a symmetric relation. Given a set of arguments $S$, an argument $\mathcal{A} \in S$ is said to be in conflict in $S$ if there is an argument $\mathcal{B} \in S$ such that $\{\mathcal{A}, \mathcal{B}\} \in \mathbf{C}$. The relation $\mathbf{R}$ is introduced in the framework and it will be used to evaluate arguments, modeling a preference criterion based on a measure of strength.

**Definition 2 (Comparison criterion)** *Given a set of arguments* $Args$*, an* argument comparison criterion $\mathbf{R}$ *is a binary relation on* $Args$*. If* $\mathcal{A}\mathbf{R}\mathcal{B}$ *but not* $\mathcal{B}\mathbf{R}\mathcal{A}$ *then* $\mathcal{A}$ *is strictly preferred to* $\mathcal{B}$*, denoted* $\mathcal{A} \succ \mathcal{B}$*. If* $\mathcal{A}\mathbf{R}\mathcal{B}$ *and* $\mathcal{B}\mathbf{R}\mathcal{A}$ *then* $\mathcal{A}$ *and* $\mathcal{B}$ *are indifferent arguments with equal relative preference, denoted* $\mathcal{A} \equiv \mathcal{B}$*. If neither* $\mathcal{A}\mathbf{R}\mathcal{B}$ *or* $\mathcal{B}\mathbf{R}\mathcal{A}$ *then* $\mathcal{A}$ *and* $\mathcal{B}$ *are incomparable arguments, denoted* $\mathcal{A} \bowtie \mathcal{B}$*.*

For two arguments $\mathcal{A}$ and $\mathcal{B}$ in $Args$, such that the pair $\{\mathcal{A}, \mathcal{B}\}$ belongs to $\mathbf{C}$ the relation $\mathbf{R}$ is considered, in order to elucidate the conflict. Depending on the preference order, two main notions of argument defeat are derived.

**Definition 3 (Defeaters)** *Let* $\Phi$=$\langle Args, \sqsubseteq, \mathbf{C}, \mathbf{R}\rangle$ *be an* $EAF$ *and let* $\mathcal{A}$ *and* $\mathcal{B}$ *be two arguments such that* $(\mathcal{A}, \mathcal{B}) \in \mathbf{C}$*. If* $\mathcal{A} \succ \mathcal{B}$ *then it is said that* $\mathcal{A}$ *is a proper defeater of* $\mathcal{B}$*. If* $\mathcal{A} \equiv \mathcal{B}$ *or* $\mathcal{A} \bowtie \mathcal{B}$*, it is said that* $\mathcal{A}$ *is a blocking defeater of* $\mathcal{B}$*, and viceversa. An argument* $\mathcal{B}$ *is said to be a defeater of an argument* $\mathcal{A}$ *if* $\mathcal{B}$ *is a blocking or a proper defeater of* $\mathcal{A}$*.*

**Example 1** *Let* $\Phi_1 = \langle Args, \sqsubseteq, \mathbf{C}, \mathbf{R}\rangle$ *be an* $EAF$ *where* $Args = \{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}\}$*,* $\sqsubseteq = \emptyset$*,* $\mathbf{C} = \{\{\mathcal{A}, \mathcal{B}\}, \{\mathcal{B}, \mathcal{C}\}, \{\mathcal{C}, \mathcal{D}\}, \{\mathcal{C}, \mathcal{E}\}\}$ *and* $\mathcal{A} \succ \mathcal{B}, \mathcal{B} \succ \mathcal{C}$*,* $\mathcal{E} \bowtie \mathcal{C}$*,* $\mathcal{C} \equiv \mathcal{D}$*. Argument* $\mathcal{A}$ *is a proper defeater of* $\mathcal{B}$*, arguments* $\mathcal{E}$ *and* $\mathcal{C}$ *are blocking each other. The same is true for* $\mathcal{C}$ *and* $\mathcal{D}$*.*

Several semantic notions were defined for EAFs. In [10], a set of accepted arguments is captured through a fixpoint operator based on the identification of *suppressed arguments*. In [12], an extension based on proof restrictions (*progressive argumentation lines*) is introduced. Semantic notions regarding strength of defenses are discussed in [13], which leads to this present work.

In the next section, the classic acceptability notion is applied to extended abstract frameworks, in order to analyze the composition of inner defenses.

## 3. Admissibility

Argumentation semantics is about argument classification through several rational positions of acceptance. A central notion in most argument extensions is *acceptability*. A very simple definition of acceptability in extended abstract frameworks is as follows.

**Definition 4 (Acceptability in EAF)** *Let* $\Phi = \langle Args, \sqsubseteq, \mathbf{C}, \mathbf{R} \rangle$ *be an EAF. An argument* $\mathcal{A} \in Args$ *is acceptable with respect to a set of arguments* $S \subseteq Args$ *if and only if every defeater* $\mathcal{B}$ *of* $\mathcal{A}$ *has a defeater in* $S$.

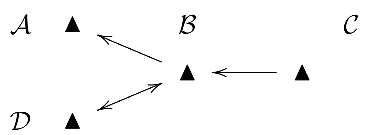


**Figure 2.** Simple extended abstract framework

Defeaters mentioned in Definition 4 may be either proper or blocking ones. In the Figure 2, argument $\mathcal{A}$ is acceptable with respect to $\{\mathcal{C}\}$,$\{\mathcal{D}\}$ and of course $\{\mathcal{C}, \mathcal{D}\}$ . Argument $\mathcal{C}$ is acceptable with respect to the empty set. Argument $\mathcal{D}$ is acceptable with respect to $\{\mathcal{C}\}$ and also with respect to $\{\mathcal{D}\}$.

Following the usual steps in argumentation semantics, the notion of acceptability leads to the notion of admissibility. This requires the definition of conflict-free set of arguments. A set of arguments $S \subseteq Args$ is said to be *conflict-free* if for all $\mathcal{A}, \mathcal{B} \in S$ it is not the case that $\{\mathcal{A}, \mathcal{B}\} \in \mathbf{C}$.

**Definition 5 (Admissible set)** *[9] Let* $\Phi = \langle Args, \sqsubseteq, \mathbf{C}, \mathbf{R} \rangle$ *be an EAF. A set of arguments* $S \subseteq Args$ *is said to be admissible if it is conflict-free and every argument in* $S$ *is acceptable with respect to* $S$.

An admissible set is able to defend any argument included in that set. Note that any blocked argument has an intrinsic self-defense due to the symmetry of blocking defeat. In [13] a restricted notion of admissibility is presented, called *x-admissibility*, where an argument is required to be defended by other arguments. Arguments that cannot be defended but by themselves are not included in x-admissible extensions.

## 4. Defenses and strength

An argument $\mathcal{A}$ may be collectively defended by several sets of arguments. The final set of accepted arguments depends on the position adopted by the rational agent, for which the knowledge is modeled by the framework. Several extensions are proposed to address this issue, as in [7,8,4,6]. An interesting position, suitable for EAFs, is to focus the acceptance in sets of strongly defended arguments. This corresponds to an agent that, given a special situation where arguments are defended in different manners, decides to accept a set where, if possible, the strongest defense is available. This is of special interest when a lot of equivalent-in-strength or incomparable arguments are involved in the defeat scenario.

This evaluation of defenses can be achieved by considering a rational, implicit ordering of argument preferences. We previously suggested this order, stating that the best defense is achieved through proper defeaters. If this is not the case, then at least is desirable a defense strong enough to block the attacks, and this is done at best by equivalent-in-strength defeaters. The worst case is to realize that both arguments, the attacker and the defender, are not related enough to evaluate a difference in force, leading to the weakest form of defeat, where the basis is only the underlying conflict. This is formalized in the next definition.

**Definition 6** *Let $\Phi = \langle Args, \sqsubseteq, \mathbf{C}, \mathbf{R} \rangle$ be an EAF. Let $\mathcal{A}$ and $\mathcal{B}$ be two arguments in $Args$. The function $pref : Args \times Args \rightarrow \{0, 1, 2\}$ is defined as follows*

$$pref(\mathcal{A}, \mathcal{B}) = \begin{cases} 0 & \textit{if } \mathcal{A} \bowtie \mathcal{B} \\ 1 & \textit{if } \mathcal{A} \equiv \mathcal{B} \\ 2 & \textit{if } \mathcal{A} \succ \mathcal{B} \end{cases}$$

Definition 6 serves as a mean to compare individual defenses. For an argument $\mathcal{A}$, the strength of its defenders is evaluated as stated in the following definition.

**Definition 7 (Defender's strength)** *Let $\Phi = \langle Args, \sqsubseteq, \mathbf{C}, \mathbf{R} \rangle$ be an EAF. Let $\mathcal{A} \in Args$ be an argument with defeater $\mathcal{B}$, which is defeated, in turn, by arguments $\mathcal{C}$ and $\mathcal{D}$. Then*

1. *$\mathcal{C}$ and $\mathcal{D}$ are equivalent in force defenders of $\mathcal{A}$ if $pref(\mathcal{C}, \mathcal{B}) = pref(\mathcal{D}, \mathcal{B})$.*
2. *$\mathcal{C}$ is a stronger defender than $\mathcal{D}$ if $pref(\mathcal{C}, \mathcal{B}) > pref(\mathcal{D}, \mathcal{B})$. It is also said that $\mathcal{D}$ is a weaker defender than $\mathcal{C}$.*

In the airline example, the argument $Fly_2$ is a stronger defender of $Fly_1$ than $Fly_3$. The evaluation of a collective defense follows from Definition 7, considering set of arguments acting as defenders.

**Definition 8 (Stronger Defense)** *Let $\Phi = \langle Args, \sqsubseteq, \mathbf{C}, \mathbf{R} \rangle$ be an EAF. Let $\mathcal{A} \in Args$ be an argument acceptable with respect to $S_1 \subseteq Args$. A set of arguments $S_2 \subseteq Args$ is said to be a stronger collective defense of $\mathcal{A}$ if $\mathcal{A}$ is acceptable with respect to $S_2$, and*

1. *There are no two arguments $\mathcal{X} \in S_1$ and $\mathcal{Y} \in S_2$ such that $\mathcal{X}$ constitutes a stronger defense than $\mathcal{Y}$*
2. *For at least one defender $\mathcal{X} \in S_1$ of $\mathcal{A}$, there exists an argument $\mathcal{Y} \in S_2 - S_1$ which constitutes a stronger defense of $\mathcal{A}$.*

A set of arguments $S_2$ is a stronger collective defense of $\mathcal{A}$ than the set $S_1$ if the force of defense achieved by elements in $S_2$ is stronger than those in $S_1$ in at most one defender, being the rest equivalent in force. Thus, every argument in $S_1$ has a correlative in $S_2$ that is a stronger or equivalent in force defender. The improvement in defense must occur through at least one new argument.

The strength of a defense is a pathway to analyze the structure of admissible sets. In extended abstract frameworks, defeat may occur in different ways, according to preference $\mathbf{R}$, and this can be used to evaluate the inner composition of an admissible set.

**Example 2** *Consider the EAF of Figure 3. The admissible sets are $\emptyset$ (trivial), every singleton set, $\{\mathcal{A}, \mathcal{D}\}$ and $\{\mathcal{A}, \mathcal{C}\}$. Argument $\mathcal{A}$ is defended by sets $\{\mathcal{D}\}$ and $\{\mathcal{C}\}$, but the first one is a stronger collective defense than the second one. Then $\{\mathcal{A}, \mathcal{D}\}$ is an admissible set with stronger inner defenses than $\{\mathcal{A}, \mathcal{C}\}$*

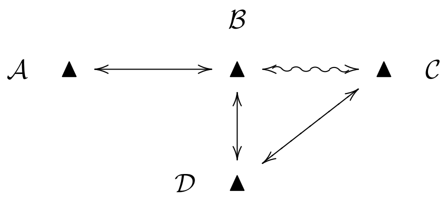


**Figure 3.** $\{\mathcal{D}\}$ is a stronger defense of $\mathcal{A}$ than $\{\mathcal{C}\}$.

The notion of strong admissible sets regarding inner defenses is captured in the following definition.

**Definition 9 (Top-admissibility)** *An admissible set of arguments $S$ is said to be top-admissible if, for any argument $\mathcal{A} \in S$, no other admissible set $S'$ includes a stronger defense of $\mathcal{A}$ than $S$.*

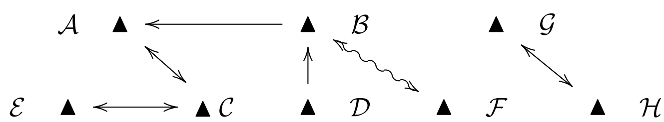


**Figure 4.** Argument defense

What top-admissibility semantics requires is that best admissible sets are selected, according to the strength of every defense. In Figure 4, the set $\{\mathcal{A}, \mathcal{D}, \mathcal{E}\}$ is top-admissible, while the set $\{\mathcal{A}, \mathcal{E}, \mathcal{F}\}$ is not. The sets $\{\mathcal{G}\}$ and $\{\mathcal{H}\}$ are also top-admissible. In the EAF of Figure 3, the set $\{\mathcal{A}, \mathcal{D}\}$ is top-admissible.

The formalization of strong defended arguments needs the identification of the precise set of defenders. This is called an *adjusted defense*.

**Definition 10 (Adjusted defense)** *Let $\Phi = \langle Args, \sqsubseteq, \mathbf{C}, \mathbf{R} \rangle$ be an EAF. Let $S$ be a set of arguments and let $\mathcal{A} \in Args$ be an argument acceptable with respect to $S$. An argument $\mathcal{B} \in S$ is a superfluous defender of $\mathcal{A}$ in $S$, if $\mathcal{A}$ is acceptable with respect to*

$S - \{\mathcal{B}\}$. *If no argument in* $S$ *is a superfluous-defender, then the set* $S$ *is said to be an adjusted defense of* $\mathcal{A}$.

In Figure 4, argument $\mathcal{A}$ is acceptable with respect to, for instance, $S = \{\mathcal{A}, \mathcal{D}, \mathcal{F}\}$. However, $S$ is not an adjusted defense of $\mathcal{A}$, as this argument is also acceptable with respect to subsets $S_1 = \{\mathcal{A}, \mathcal{D}\}$ and $S_2 = \{\mathcal{A}, \mathcal{F}\}$, being these sets adjusted defenses for $\mathcal{A}$. As it is free of defeaters, argument $\mathcal{D}$ has the empty set as the only adjusted defense, while the set $\{\mathcal{G}\}$ is an adjusted defense of $\mathcal{G}$. The same is true for $\mathcal{H}$.

Note that it is possible for an argument to have more than one adjusted defense. In fact, if an argument $\mathcal{X}$ is involved in a blocking defeat, it can be included in an adjusted defense for $\mathcal{X}$, since it is able to defend itself against non-preferred defeaters.

**Definition 11 (Dead-end defeater)** *An argument* $\mathcal{B}$ *is said to be a dead-end defeater of an argument* $\mathcal{A}$ *if the only defense of* $\mathcal{A}$ *against* $\mathcal{B}$ *is* $\mathcal{A}$ *itself.*

Dead-end defeaters arise only when an argument $\mathcal{A}$ is involved in a blocking situation with another argument without third defeaters, and therefore $\mathcal{A}$ cannot appeal to other defenses on that attack. This leads to self-defender arguments.

**Definition 12 (Weak-acceptable)** *An argument* $\mathcal{A}$ *is said to be a self-defender if for every adjusted defense* $S$ *of* $\mathcal{A}$*, then* $\mathcal{A} \in S$*. In that case,* $\mathcal{A}$ *is said to be weak-acceptable with respect to* $S$ *if (i)* $|S| > 1$*, and (ii)* $\mathcal{A}$ *is defended by* $S - \{\mathcal{A}\}$ *against every non dead-end defeater.*

A self-defender argument $\mathcal{A}$ is considered weak-acceptable with respect to a set $S$ if it is actually defended by $S$ from other attacks. If $|S| > 1$ then clearly $\mathcal{A}$ cannot be defended by itself against all its defeaters. Weak-acceptability requires $\mathcal{A}$ to be defended whenever other defenders are available. This is because $\mathcal{A}$, being enforced to be a self-defeater, may be also defending itself against non dead-end defeaters.

In Figure 4, arguments $\mathcal{G}$ and $\mathcal{H}$ are self-defender arguments. Argument $\mathcal{C}$ is weak-acceptable with respect to $\{\mathcal{C}, \mathcal{B}\}$. Although $\mathcal{C}$ is acceptable with respect to $\{\mathcal{C}\}$, it is not weak-acceptable with respect to that set, as every attack requires the defense of $\mathcal{C}$. Self-defender arguments are relevant to elaborate a more flexible notion of acceptance. This will be addressed in Section 6.

Clearly, an argument $\mathcal{A}$ may have a set of adjusted defenses. When $\mathcal{A}$ is included in an admissible set $S$, then some or all of these adjusted defenses are included in $S$. Even more, the intersection of all adjusted defenses of $\mathcal{A}$ is included in $S$.

**Proposition 1** *Let* $D = \{S_1, S_2, ..., S_n\}$ *be the set of all adjusted defenses of an argument* $\mathcal{A}$*. For every admissible set* $T$ *such that* $\mathcal{A} \in T$*, the following holds:*

1. $S_i \subseteq T$*, for some* $i$*,* $1 \leq i \leq n$*.*
2. $\bigcap_{i=1}^{i=n} S_i \subseteq T$

*Proof*:

1. Argument $\mathcal{A}$ is acceptable with respect to $T$, and therefore $T$ is a collective defense for $\mathcal{A}$. Let $V = \{X_1, X_2, ..., X_m\}$ be the set of defeaters of $\mathcal{A}$. Every element of $V$ is defeated by an argument in $T$. Let $W \subseteq T$ be a minimal set of arguments such that every element in $W$ is a defeater of an argument in $V$. Then

the set $\mathcal{A}$ is acceptable with respect to $W$. As $W$ is minimal, it is an adjusted defense, and therefore $W = S_i$ for some $i$, $1 \leq i \leq n$.

2. Trivial from previous proof. □.

Adjusted defenses are a guarantee on acceptance. If every argument in at least one adjusted defense is accepted, then the defended argument may also be included in the acceptance set. Definition 8 can be used to compare defenses.

**Definition 13 (Forceful defense)** *Let $\Phi = \langle Args, \sqsubseteq, \mathbf{C}, \mathbf{R} \rangle$ be an EAF. Let $S$ be a set of arguments and let $\mathcal{A} \in Args$. The set $S$ is a forceful-defense of $\mathcal{A}$ if $S$ is an adjusted defense of $\mathcal{A}$ and no other adjusted defense is a stronger defense than $S$.*

Following the example in Figure 4, the sets $S_1 = \{\mathcal{A}, \mathcal{D}\}$ and $S_2 = \{\mathcal{A}, \mathcal{F}\}$ are adjusted defenses of $\mathcal{A}$. However, $S_1$ is a stronger collective defense of $\mathcal{A}$ than $S_2$. Therefore, $S_1$ is a forceful-defense of $\mathcal{A}$. Note that this kind of tightened defense is not unique: the set $S_3 = \{\mathcal{E}, \mathcal{D}\}$ is also a forceful-defense. Forceful defense is ideal in the sense that the strongest defenders are used, and therefore is a measure of quality for argument acceptance. An argument accepted by the use of a forceful-defense is strongly endorsed in the acceptance set.

**Definition 14 (Forceful argument inclusion)** *Let $\Phi = \langle Args, \sqsubseteq, \mathbf{C}, \mathbf{R} \rangle$ be an EAF. Let $S$ be an admissible set of arguments and let $\mathcal{A} \in S$. The argument $\mathcal{A}$ is said to be forcefully-included in $S$ if at least one forceful-defense of $\mathcal{A}$ is included in $S$.*

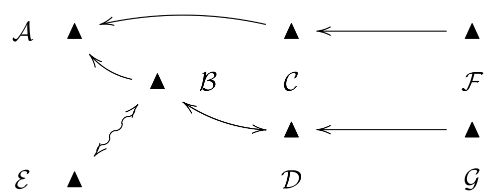


**Figure 5.** $\mathcal{A}$ is not forcefully-included in $\{\mathcal{A}, \mathcal{E}, \mathcal{F}, \mathcal{G}\}$

**Example 3** *Consider the EAF of Figure 5. In the admissible set $S = \{\mathcal{A}, \mathcal{E}, \mathcal{F}, \mathcal{G}\}$, the argument $\mathcal{A}$ is not forcefully-included in $S$ because the adjusted defense of $\mathcal{A}$ in that set is $\{\mathcal{E}, \mathcal{F}\}$, which is not the strongest collective defense. Note that $\{\mathcal{D}, \mathcal{F}\}$ is the forceful-defense of $\mathcal{A}$, as $\mathcal{D}$ is stronger defender than $\mathcal{E}$. However, $\mathcal{D}$ is not included in $S$ and therefore $\mathcal{A}$ cannot be reinstated by the use of its strongest defender but $\mathcal{E}$.*

Forceful inclusion requires the use of the strongest adjusted defenses. This resembles top-admissibility, although there is a difference in strength requirement.

**Proposition 2** *Let $\Phi = \langle Args, \sqsubseteq, \mathbf{C}, \mathbf{R} \rangle$ be an EAF. Let $S$ be an admissible set of arguments. If every argument in $S$ is forcefully-included in $S$, then $S$ is top-admissible.*

*Proof*: If every argument $\mathcal{X}$ in $S$ is forcefully included in $S$, then every strongest defense of $\mathcal{X}$ is included in $S$ and therefore no other set provides a stronger defense of $\mathcal{X}$. Then

$S$ is top-admissible. $\square$.

The converse of Proposition 2, however, is not true. In Figure 5 the set $\{\mathcal{A}, \mathcal{E}, \mathcal{F}, \mathcal{G}\}$ is top-admissible, even though argument $\mathcal{A}$ is not forcefully-included in that set. In the following section these notions are studied in the context of Dung's grounded extension.

## 5. Grounded extension

In Dung's work, a skeptical position of argument acceptance in argumentation frameworks is captured by the grounded extension.

**Definition 15 (Grounded extension)** *[9] An admissible set $R$ of arguments is a complete extension if and only if each argument which is acceptable with respect to $R$, belongs to $R$. A set of arguments $G$ is a grounded extension if and only if it is the least (with respect to set inclusion) complete extension.*

The grounded extension is unique and it captures the arguments that can be directly or indirectly defended by defeater-free arguments. This extension is also the least fixpoint of a simple monotonic function:

$$F_{AF}(S) = \{\mathcal{A} : \mathcal{A} \text{ is acceptable with respect to } S\}.$$

The following theorem states that this skeptical position is also based on strong defense.

**Theorem 5.1** *Let $\Phi = \langle Args, \sqsubseteq, \mathbf{C}, \mathbf{R}\rangle$ be an EAF. Let $GE$ be the grounded extension of $\Phi$. Then every argument in $GE$ is forcefully-included in $GE$.*

*Proof*: The set $F_\Phi(\emptyset)$ is formed by all defeater-free arguments in $Args$. These arguments cannot be involved in a blocking defeat as they lack of defeaters. The strongest adjusted defense for every argument in $F_\Phi(\emptyset)$ is the empty set and thus they are forcefully included in $F_\Phi(\emptyset)$. Because no blocked defeat is present, any defense these arguments provide is through proper defeat, which is the strongest defense. Every argument acceptable with respect to $F_\Phi(\emptyset)$ has a strongest adjusted defense on that set and then it is forcefully included in $F^2_\Phi(\emptyset)$. Suppose every argument in $F^k_\Phi(\emptyset)$ is forcefully-included in $F^k_\Phi(\emptyset)$. We will prove that an argument acceptable with respect to $F^k_\Phi(\emptyset)$ is forcefully included in $F^{k+1}_\Phi(\emptyset)$. Let $\mathcal{A} \in Args - F^k_\Phi(\emptyset)$ acceptable with respect to $F^k_\Phi(\emptyset)$ and let $\mathcal{B}$ a defeater of $\mathcal{A}$. Let $\mathcal{C} \in F^k_\Phi(\emptyset)$ be a defender of $\mathcal{A}$ against $\mathcal{B}$.

- if $\mathcal{C}$ is a proper defeater of $\mathcal{B}$, then $\mathcal{C}$ is a strongest defense with respect to $\mathcal{B}$, favoring the forceful inclusion of $\mathcal{A}$.
- if $\mathcal{C}$ is a blocking defeater of $\mathcal{B}$, then $\mathcal{C}$ as part of $F^k_\Phi(\emptyset)$ was previously defended from $\mathcal{B}$ by an argument $\mathcal{D} \in F^k_\Phi(\emptyset)$. By hypothesis, $\mathcal{C}$ is forcefully included in $F^k_\Phi(\emptyset)$ and then $\mathcal{D}$ is a proper defeater of $\mathcal{B}$. Thus, $\mathcal{D}$ is a strongest defense with respect to $\mathcal{B}$, favoring the forceful inclusion of $\mathcal{A}$.

As in any case $\mathcal{A}$ has a strongest defense in $F^k_\Phi(\emptyset)$, then $\mathcal{A}$ is forcefully included in $\mathcal{D} \in F^{k+1}_\Phi(\emptyset)$. $\square$.

Because of Theorem 5.1, in Figure 5 it is clear that argument $\mathcal{A}$ is not in the grounded extension, as it is not forcefully-included in any admissible set. If the argumentation framework is uncontroversial [9], then the grounded extension is the intersection of all preferred extensions. As a consequence, the skeptical acceptance with respect to preferred semantics requires dropping out every non forcefully-included argument.

## 6. Weak grounded extension

The grounded extension only captures forcefully-included arguments. It is possible to adopt a more credulous approach, expanding the acceptance by considering self-defender arguments.

**Definition 16** *Let* $\Phi$=$\langle Args, \sqsubseteq, \mathbf{C}, \mathbf{R}\rangle$*be an EAF. The extended characteristic function of* $\Phi$ *is defined as* $F^{\cup}_{\Phi}(S) = F_{\Phi}(S) \cup \{\mathcal{A} : \mathcal{A}$ *is weak acceptable with respect to* $S \cup \mathcal{A}\}$

The function $F^{\cup}_{\Phi}$ is more permissive than classic characteristic function in the sense that it allows the inclusion of self-defender arguments whenever they are partially defended by arguments in $S$.

**Proposition 3** *If* $S$ *is an admissible set of arguments, then* $F^{\cup}_{\Phi}(S)$ *is admissible.*

*Proof*: Clearly, if $S$ is admissible, then $F_{\Phi}(S)$ is also admissible [9]. We are going to proof that the addition of weak acceptable arguments does not disrupt the admissibility notion. Suppose $F^{\cup}_{\Phi}(S)$ is not admissible. Then either (a) an argument is not acceptable with respect to that set, or (b) it is not conflict-free:
(a) Suppose there is an argument $\mathcal{B} \in F^{\cup}_{\Phi}(S)$ such that $\mathcal{B}$ is not acceptable with respect to $F^{\cup}_{\Phi}(S)$. As $F_{\Phi}(S)$ is admissible, then clearly $\mathcal{B}$ is weak-acceptable with respect to $S \cup \{\mathcal{B}\}$ (it cannot belong to $F_{\Phi}(S)$). Being a self-defender argument, some of the defeaters of $\mathcal{B}$ are defeated by $S$, while the rest is defeated by $\mathcal{B}$ itself. Thus, every defeater of $\mathcal{B}$ has a defeater in $S \cup \{\mathcal{B}\} \subseteq F^{\cup}_{\Phi}(S)$. But then $\mathcal{B}$ is acceptable with respect to $F^{\cup}_{\Phi}(S)$, which is a contradiction.
(b) Suppose there are two arguments $\mathcal{A}$, $\mathcal{B}$ in $F^{\cup}_{\Phi}(S)$ such that $\{\mathcal{A}, \mathcal{B}\} \in \mathbf{C}$. If, say, $\mathcal{A}$ is a proper defeater of $\mathcal{B}$, then there exists an argument $\mathcal{C}$ in $S$ such that $\mathcal{C}$ defeats $\mathcal{A}$ ($\mathcal{A}$ is not a dead-end defeater of $\mathcal{B}$ and it is required to be defeated by $S$). But then $\mathcal{A}$ is not acceptable with respect to $S$ and it can only be weak acceptable with respect to $S \cup \{\mathcal{A}\}$. Now $\mathcal{C}$ cannot be a dead-end defeater of $\mathcal{A}$ (it should not be in $S$ then) and then $S$ defends $\mathcal{A}$ against $\mathcal{C}$, thus $S$ is not conflict free, which is a contradiction. On the other hand, if $\mathcal{A}$ and $\mathcal{B}$ are blocking defeaters then at least on them, say $\mathcal{A}$, is a self-defender argument. Then $\mathcal{B}$ cannot be its dead-end defeater (otherwise it is excluded) and then an argument $\mathcal{D} \in S$ defeats $\mathcal{B}$ ($S$ defends $\mathcal{A}$). But then $\mathcal{B}$ is not acceptable with respect to $S$, and it must be also a self-defender argument. Thus, it is defended by $S$ of every non dead-end defeater. In particular, $\mathcal{B}$ is defended against $\mathcal{D}$ by an argument $\mathcal{C} \in S$. But then $S$ is not conflict-free, which is a contradiction.
As suppositions (a) and (b) lead to contradiction, then $F^{\cup}_{\Phi}(S)$ is admissible. $\square$

Following the same steps in [9], an extension can be obtained using function $F^{\cup}_{\Phi}$. This function is monotonic (wrt set inclusion), because an argument that is weak-acceptable with respect to $S$ is also weak-acceptable with respect to supersets of $S$.

**Definition 17** *Let* $\Phi$=$\langle Args, \sqsubseteq, \mathbf{C}, \mathbf{R}\rangle$*be an EAF. The weak grounded extension is the least fixpoint of function* $F_\Phi^\cup(S)$*.*

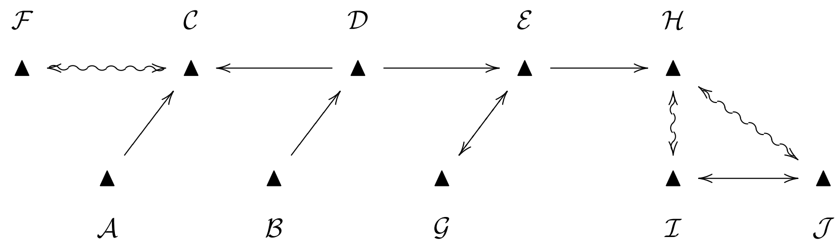


**Figure 6.** $\{\mathcal{A}, \mathcal{B}, \mathcal{F}, \mathcal{E}\}$ is the weak grounded extension

**Example 4** *Consider the EAF of Figure 6. The grounded extension is* $\{\mathcal{A}, \mathcal{B}, \mathcal{F}\}$*. The weak grounded extension is* $\{\mathcal{A}, \mathcal{B}, \mathcal{F}, \mathcal{E}\}$*, because argument* $\mathcal{E}$ *is weak-acceptable with respect to* $\{\mathcal{B}, \mathcal{E}\}$*. Arguments* $\mathcal{I}$ *and* $\mathcal{J}$ *are not included in the weak grounded extension because they are not self-defenders. For example,* $\{\mathcal{H}, \mathcal{J}\}$ *is an adjusted defense for* $\mathcal{I}$*. Clearly, there is a contradiction between these three arguments as the adjusted defense is not conflict free. In fact, any cycle of blocking defeaters is naturally formed by non self-defender arguments.*

**Example 5** *In the framework of Figure 5 the weak grounded extension is* $\{\mathcal{F}, \mathcal{G}, \mathcal{B}\}$*. In the framework of Figure 4 the grounded extension and the weak grounded extension coincides, as the only argument in a weak-acceptability condition is* $\mathcal{C}$*, but is not able to be defended from* $\mathcal{A}$ *by arguments other than itself.*

The addition of arguments under a weak acceptability condition is harmless in the sense of including only arguments partially defended by an admissible set. For the rest of the attacks, the argument itself is enough as defense. This allows the further inclusion of arguments that cannot be considered in the classic grounded extension.

## 7. Conclusions

In this paper we analyzed the strength of defenses in extended argumentation frameworks. In this formalism, the defeat relation is derived using a preference criterion over arguments. Thus, a *proper defeat* is consequence of preferring an argument in a conflicting pair, while a *blocking defeat* is consequence of incomparable or equivalent-in-force conflicting arguments. The defense of an argument may be achieved by proper defeaters or by blocking defeaters. Clearly, the quality of a defense depends on the type of defeaters used. Defense through proper defeaters is stronger than defense through blocking defeaters. Even more, in blocking situations, the defense provided by equivalent in force arguments may be considered stronger than the defense provided by incomparable arguments. Under this position, the force of a defense is formalized, and it is possible to evaluate how well defended an argument is when included in an admissible set. An argument is forcefully included in an admissible set when the best defense is captured by that set. A top-admissible set is including, for every argument in the set, the strongest defense as it is possible to conform admissibility. In extended abstract frameworks, every argument in the classic grounded extension is forcefully included in that set, and then arguments with weaker defenses are dropped out. In order to adopt a slightly credulous approach, the notion of *weak acceptability* is introduced, allowing the definition of the *weak grounded extension*, where arguments can partially defend themselves. The future works will be the study of the relation between weak grounded extension and warrant extension for EAFs as defined in [12], and other semantic notions regarding blocking defeaters, as *x-admissibility*, which was previous presented in [13].

## 8. Acknowledgments

The authors would like to thank anonymous referees for comments that led to substantial improvements and error corrections regarding weak acceptability.

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