Progressive Defeat Paths in Abstract Argumentation Frameworks

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Abstract. Abstract argumentation systems are formalisms for defeasible reasoning where some components remain unspecified, the structure of arguments being the main abstraction. In the dialectical process carried out to identify accepted arguments in the system some controversial situations may appear. These relate to the reintroduction of arguments into the process which cause the onset of circularity. This must be avoided in order to prevent an infinite analysis. Some systems apply the sole restriction of not allowing the introduction of previously considered arguments in an argumentation line. However, repeating an argument is not the only possible cause for the risk mentioned. A more specific restriction needs to be applied considering the existence of subarguments. In this work, we introduce an extended argumentation framework where two kinds of defeat relation are present, and a definition for *progressive defeat path*.

1 Introduction

Different formal systems of defeasible argumentation have been defined as forms of representing interesting characteristics of practical or common sense reasoning. The central idea in these systems is that a proposition will be accepted if there exists an argument that supports it, and this argument is regarded as acceptable with respect to an analysis performed considering all the available counterarguments. Therefore, in the set of arguments of the system, some of them will be *acceptable* or *justified* or *warranted* arguments, while others will be not. In this manner, defeasible argumentation allows reasoning with incomplete and uncertain information and is suitable to handle inconsistency in knowledge-based systems.

Abstract argumentation systems [1, 3, 12] are formalisms for defeasible reasoning where some components remain unspecified, being the structure of arguments the main abstraction. In this type of systems, the emphasis is put on elucidating semantic questions, such as finding the set of accepted arguments. Most of them are based on the single abstract notion of *attack* represented as a relation among the set of available arguments. From that relation, several *argument extensions* are defined as sets of possible accepted arguments.

For example, the argumentation framework defined by Dung in [1] is a pair $(AR, attacks)$, where AR is a set of arguments, and $attacks$ is a binary relation on AR, i.e. $attacks \subseteq AR \times AR$. In Dung's approach several semantic notions are defined as argument extensions. For example, a set of arguments S is said to be *conflict-free* if there are no arguments $\mathcal{A}, \mathcal{B}$ in S such that $\mathcal{A}$ attacks $\mathcal{B}$. The set of accepted arguments is characterized using the concept of *acceptability.* An argument $\mathcal{A} \in AR$ is *acceptable* with respect to a set of arguments S if and only if every argument $\mathcal{B}$ attacking $\mathcal{A}$ is attacked by an argument in S. It is also said that S is defending $\mathcal{A}$ against its attackers, and this is a central notion on argumentation. A set R of arguments is a *complete extension* if R defends every argument in R. A set of arguments G is a *grounded extension* if and only if it is the least (with respect to set inclusion) complete extension. The grounded extension is also the least fixpoint of a simple monotonic function:

$$F_{AF}(S) = \{\mathcal{A} : \mathcal{A} \text{ is acceptable wrt } S\}.$$

In [1], theorems stating conditions of existence and equivalence between these extensions are also introduced.

Although the area of abstract argumentation has greatly evolved, the task of comparing arguments to establish a preference is not always successful. Having a preference relation in the set of arguments is essential to determine a defeat relation. In [5], an abstract framework for argumentation with two types of argument defeat relation are defined among arguments. In the dialectical process carried out to identify accepted arguments in the system, some controversial situations may be found, as previously presented in [10, 2]. These situations are related to the reintroduction of arguments in this process, causing a circularity that must be avoided in order to prevent an infinite analysis. Consider for example three arguments $\mathcal{A}, \mathcal{B}$ and $\mathcal{C}$ such that $\mathcal{A}$ is a defeater of $\mathcal{B}$, $\mathcal{B}$ is a defeater of $\mathcal{C}$ and $\mathcal{C}$ is a defeater of $\mathcal{A}$. In order to decide the acceptance of $\mathcal{A}$, the acceptance of its defeaters must be analyzed first, including $\mathcal{A}$ itself.

An *argumentation line* is a sequence of defeating arguments, such as $[\mathcal{A}, \mathcal{B}]$ or $[\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{A}]$ in the system above. Whenever an argument $\mathcal{A}$ is encountered while analyzing arguments for and against $\mathcal{A}$, a circularity occurs. Some systems apply a single restriction to argumentation lines: no previously considered argument is reintroduced in the process. In [10], the relation between circularity in argumentation and the comparison criteria used in the system is established. Arguments in such situations are called *fallacious arguments* and the circularity itself is called a *fallacy.* In somes systems such as [3, 4], these arguments are classified as *undecided* arguments: they are not accepted nor rejected.

In this work, we show that a more specific restriction needs to be applied, other than to the prohibit reintroduction of previous arguments in argumentation lines. In the next section, we define the extended abstract framework in order to characterize *progressive* argumentation lines.

2 Abstract Argumentation Framework

Our abstract argumentation framework is formed by four elements: a set of arguments, the subargument relation, a binary conflict relation over this set, and a function used to decide which argument is preferred given any pair of arguments.

Definition 1. *An abstract argumentation framework is a quartet $\langle AR, \sqsubseteq, \mathbf{C}, \pi \rangle$, where AR is a finite set of arguments, $\sqsubseteq$ is the subargument relation, $\mathbf{C}$ is a symmetric and anti-reflexive binary conflict relation between arguments, $\mathbf{C} \subseteq AR \times AR$, and $\pi : AR \times AR \longrightarrow 2^{AR}$ is a preference function among arguments.*

Here, arguments are abstract entities [1] that will be denoted using calligraphic uppercase letters. No reference to the underlying logic is needed since we are abstracting the structure of the arguments (see [6, 11, 8, 9, 2] for concrete systems). The symbol $\sqsubseteq$ denotes subargument relation: $\mathcal{A} \sqsubseteq \mathcal{B}$ means "$\mathcal{A}$ is a subargument of $\mathcal{B}$". Any argument $\mathcal{A}$ is considered a superargument and a subargument of itself. Any subargument $\mathcal{B} \sqsubseteq \mathcal{A}$ such that $\mathcal{B} \neq \mathcal{A}$ is said to be a non-trivial subargument. Non-trivial subargument relation is denoted by symbol $\sqsubset$. The following notation will be also used: given an argument $\mathcal{A}$ then $\mathcal{A}^-$ will represent a subargument of $\mathcal{A}$, and $\mathcal{A}^+$ will represent a superargument of $\mathcal{A}$. When no confusion may arise, subscript index will be used for distinguishing different subarguments or superarguments of $\mathcal{A}$.

Example 1. Let $\Phi = \langle AR, \sqsubseteq, \mathbf{C}, \pi \rangle$ be an argumentation framework, where: $AR = \{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}\}$, $\mathcal{B} \sqsubseteq \mathcal{A}, \mathcal{D} \sqsubseteq \mathcal{C}$, $\mathbf{C} = \{\{\mathcal{C}, \mathcal{B}\}, \{\mathcal{C}, \mathcal{A}\}, \{\mathcal{E}, \mathcal{D}\}, \{\mathcal{E}, \mathcal{C}\}\}$[1], $\pi(\mathcal{C}, \mathcal{B}) = \{\mathcal{C}\}$, and $\pi(\mathcal{E}, \mathcal{D}) = \{\mathcal{E}\}$ [2].

The conflict relation between two arguments $\mathcal{A}$ and $\mathcal{B}$ denotes the fact that these arguments cannot be accepted simultaneously since they contradict each other. For example, two arguments $\mathcal{A}$ and $\mathcal{B}$ that support complementary conclusions cannot be accepted together. Conflict relations are denoted by unordered pairs of arguments, and the set of all pairs of arguments in conflict on Φ is denoted by $\mathbf{C}$. Given a set of arguments S, an argument $\mathcal{A} \in S$ is said to be in conflict in S if there is an argument $\mathcal{B} \in S$ such that $(\mathcal{A}, \mathcal{B}) \in \mathbf{C}$. Given an argument $\mathcal{A}$ we define $Conf(\mathcal{A})$ as the set of all arguments $\mathcal{X} \in AR$ such that $(\mathcal{A}, \mathcal{X}) \in \mathbf{C}$. As stated by the following axiom, conflict relations have to be propagated to superarguments.

Axiom 1 (Conflict inheritance) *Let $\Phi = \langle AR, \sqsubseteq, \mathbf{C}, \pi \rangle$ be an argumentation framework, and let $\mathcal{A}$ and $\mathcal{B}$ be two arguments in AR. If $\mathcal{A}$ and $\mathcal{B}$ are in conflict, then the conflict is inherited by any superargument of $\mathcal{A}$ and $\mathcal{B}$. That is, if $(\mathcal{A}, \mathcal{B}) \in \mathbf{C}$, then $(\mathcal{A}, \mathcal{B}^+) \in \mathbf{C}$, $(\mathcal{A}^+, \mathcal{B}) \in \mathbf{C}$, and $(\mathcal{A}^+, \mathcal{B}^+) \in \mathbf{C}$, for any superargument $\mathcal{A}^+$ of $\mathcal{A}$ and $\mathcal{B}^+$ of $\mathcal{B}$.*

[1] When describing elements of $\mathbf{C}$, we write $\{\mathcal{A}, \mathcal{B}\}$ as an abbreviation for $\{(\mathcal{A}, \mathcal{B}), (\mathcal{B}, \mathcal{A})\}$, for any arguments $\mathcal{A}$ and $\mathcal{B}$ in AR.

[2] Note that only the relevant cases, those involving conflicting arguments, of function π are shown.

The constraints imposed by the conflict relation lead to several sets of possible accepted arguments. For example, if $AR = \{\mathcal{A}, \mathcal{B}\}$ and $(\mathcal{A}, \mathcal{B}) \in \mathbf{C}$, then $\{\mathcal{A}\}$ is a set of possible accepted arguments, and so is $\{\mathcal{B}\}$. Therefore, some way of deciding among all the possible outcomes must be devised. In order to accomplish this task, the function π is introduced in the framework along with the set of arguments and the conflict relation. The function π will be used to evaluate arguments, comparing them under a preference criterion.

Definition 2. *Given a set of arguments AR, an argument comparison criterion is a preference function $\pi : AR \times AR \longrightarrow 2^{AR}$, and $\pi(\mathcal{A}, \mathcal{B}) \in \wp(\{\mathcal{A}, \mathcal{B}\})$.*

Remark 1 *If $\pi(\mathcal{A}, \mathcal{B}) = \{\mathcal{A}\}$ then $\mathcal{A}$ is preferred to $\mathcal{B}$. In the same way, if $\pi(\mathcal{A}, \mathcal{B}) = \{\mathcal{B}\}$ then $\mathcal{B}$ is preferred to $\mathcal{A}$. If $\pi(\mathcal{A}, \mathcal{B}) = \{\mathcal{A}, \mathcal{B}\}$ then $\mathcal{A}$ and $\mathcal{B}$ are arguments with equal relative preference. If $\pi(\mathcal{A}, \mathcal{B}) = \emptyset$ then $\mathcal{A}$ and $\mathcal{B}$ are incomparable arguments. Observe that $\pi(\mathcal{A}, \mathcal{B}) = \pi(\mathcal{B}, \mathcal{A})$.*

Given an argumentation framework $\langle AR, \sqsubseteq, \mathbf{C}, \pi \rangle$ where $\mathcal{A}$ and $\mathcal{B}$ are in AR, and $(\mathcal{A}, \mathcal{B}) \in \mathbf{C}$, according to definition 2 there are four possible outcomes:

- $\pi(\mathcal{A}, \mathcal{B}) = \{\mathcal{A}\}$. In this case a *defeat* relation is established. Because $\mathcal{A}$ is preferred to $\mathcal{B}$, in order to accept $\mathcal{B}$ it is necessary to analyze the acceptance of $\mathcal{A}$, but not the other way around. It is said that argument $\mathcal{A}$ *defeats* argument $\mathcal{B}$, and $\mathcal{A}$ is a *proper defeater* of $\mathcal{B}$.
- $\pi(\mathcal{A}, \mathcal{B}) = \{\mathcal{B}\}$. In a similar way, argument $\mathcal{B}$ *defeats* argument $\mathcal{A}$, and therefore $\mathcal{B}$ is a *proper defeater* of $\mathcal{A}$.
- $\pi(\mathcal{A}, \mathcal{B}) = \{\mathcal{A}, \mathcal{B}\}$. Both arguments are equivalent, *i.e.* there is no relative difference of conclusive force, so $\mathcal{A}$ and $\mathcal{B}$ are said to be *indistinguishable* regarding the preference relacion π. No proper defeat relation can be established between these arguments.
- $\pi(\mathcal{A}, \mathcal{B}) = \emptyset$. Both arguments are *incomparable* according to π, and no *proper* defeat relation is inferred.

In the first two cases, a concrete preference is made between two arguments, and therefore a defeat relation is established. The preferred arguments are called *proper defeaters*. In the last two cases, no preference is made, either because both arguments are indistinguishable from each other or because they are incomparable. These cases are slightly different. If the arguments are indistinguishable, then according to π they have the *same* relative conclusive force. For example, if the preference criterion establishes that smaller[3] arguments are preferred, then two arguments of the same size are indistinguishable. On the other hand, if the arguments are *incomparable* then π is not able to establish a relative difference of conclusive force. For example, if the preference criterion states that argument $\mathcal{A}$ is preferred to $\mathcal{B}$ whenever the premises of $\mathcal{A}$ are included in the premises of

[3] In general, the size of an argument may be defined on structural properties of arguments, as the number of logical rules used to derive the conclusion or the number of propositions involved in that process.

$\mathcal{B}$, then arguments with disjoint sets of premises are incomparable. This situation seems to expose a limitation of π, but must be understood as a natural behaviour. Some arguments just cannot be compared.

When two conflictive arguments are indistinguishable or incomparable, the conflict between these two arguments remains unresolved. Due to this situation and to the fact that the conflict relation is a symmetric relation, each of the arguments is *blocking* the other one and it is said that both of them are *blocking defeaters* [7, 11]. An argument $\mathcal{B}$ is said to be a *defeater* of an argument $\mathcal{A}$ if $\mathcal{B}$ is a blocking or a proper defeater of $\mathcal{A}$.

Example 2. Let $\Phi = \langle AR, \sqsubseteq, \mathbf{C}, \pi \rangle$ be an argumentation framework, where: $AR = \{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}\}$, $\mathbf{C} = \{\{\mathcal{A}, \mathcal{B}\}, \{\mathcal{B}, \mathcal{C}\}, \{\mathcal{C}, \mathcal{D}\}\}$ and $\pi(\mathcal{A}, \mathcal{B}) = \{\mathcal{A}\}, \pi(\mathcal{B}, \mathcal{C}) = \{\mathcal{B}\}$ and $\pi(\mathcal{C}, \mathcal{D}) = \{\mathcal{C}, \mathcal{D}\}$. Here, argument $\mathcal{A}$ is a proper defeater of argument $\mathcal{B}$, while $\mathcal{C}$ is a blocking defeater of $\mathcal{D}$ and vice versa, $\mathcal{D}$ is a blocking defeater of $\mathcal{C}$.

Abstract frameworks can be depicted as graphs, with different types of arcs. We use the arc ($\cdots\cdots\bullet$) to denote the subargument relation. An arrow ($\longrightarrow$) is used to denote proper defeaters and a double-pointed arrow ($\longleftrightarrow$) connects blocking defeaters. In figure 1, a simple framework is shown. Argument $\mathcal{C}$ is a subargument of $\mathcal{A}$. Argument $\mathcal{B}$ is a proper defeater of $\mathcal{C}$ and $\mathcal{D}$ is a blocking defeater of $\mathcal{B}$ and viceversa.

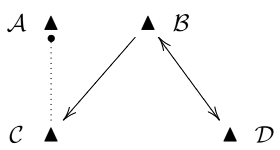

Fig. 1. Defeat graph

Some authors leave the preference criteria unspecified, even when it is one of the most important components in the system. However, in many cases it is sufficient to establish a set of properties that the criteria must exhibit. A very reasonable one states that an argument is as strong as its weakest subargument [12]. We formalize this idea in the next definition.

Definition 3 (Monotonic preference relation). *A preference relation π is said to be monotonic if, given $\pi(\mathcal{A}, \mathcal{B}) = \{\mathcal{A}\}$, then $\pi(\mathcal{A}, \mathcal{B}) = \pi(\mathcal{A}, \mathcal{B}_i^+)$, for any arguments $\mathcal{A}$ and $\mathcal{B}$ in Φ.*

We will assume from now on that the criterion π included in Φ is monotonic. This is important because any argument $\mathcal{A}$ defeated by another argument $\mathcal{B}$ should also be defeated by another argument $\mathcal{B}^+$.

In figure 2, a simple framework is depicted corresponding to example 2. Here argument $\mathcal{C}$ defeats $\mathcal{B}$, but it should also be a defeater of $\mathcal{A}$, because $\mathcal{B}$ is its subargument. The same holds for arguments $\mathcal{E}, \mathcal{C}$ and $\mathcal{D}$.

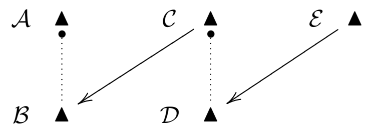

Fig. 2. An abstract framework

(a) (b)

Fig. 3. Defeating subarguments

In figure 3, argument $\mathcal{B}$ is shown defeating argument $\mathcal{A}$ via its subargument $\mathcal{A}_i$ and two valid ways to depict this situation. The arrow denoting the defeat relation between $\mathcal{B}$ and $\mathcal{A}$ as shown in (a), may be omitted if subargument arcs are drawn in the graph, as in (b).

3 Argumentation Semantics

In [1], several semantic notions are defined. Other forms of clasifying arguments as *accepted* or *rejected* can be found in [3, 4]. However, these concepts are applied to abstract frameworks with single attack relation, as the one originally shown by Dung. It is widely accepted that defeat between arguments must be defined over two basic elements: contradiction and comparison. The first one states that when two arguments are contradictory and therefore cannot be accepted simultaneously. The second one determines which of these argument is preferred to the other, using a previously defined comparison method. Due to the possibility of lack of decision at comparison stage, the outcome of this process is not always equivalent to an attack relation as in [1]. According to this situation, our framework includes two kind of relations: proper defeat and blocking defeat. We will focus in this section on the task of defining the structure of a well-formed argumentation line, from an abstract point of view.

Definition 4 (Defeat path). *A* defeat path λ *of an argumentation framework* $\langle AR, \sqsubseteq, \mathbf{C}, \pi \rangle$ *is a finite sequence of arguments* $[A_1, A_2, \ldots, A_n]$ *such that argument* A_{i+1} *is a defeater of argument* A_i *for any* $0 < i < n$*. The number of arguments in the path is denoted* $|\lambda|$*.*

A defeat path is a sequence of defeating arguments. The length of the defeat path is important for acceptance purposes, because an argument $\mathcal{A}$ defeated by an argument $\mathcal{B}$ may be reinstated by another argument $\mathcal{C}$. In this case, it is said that argument $\mathcal{C}$ *defends* $\mathcal{A}$ against $\mathcal{B}$. Note that three arguments are involved in a defense situation: the attacked, the attacker and the defender.

Definition 5 (Defeat paths for an argument). *Let $\Phi =< AR, \mathbf{C}, \pi >$ be an argumentation framework and $\mathcal{A} \in AR$. A defeat path for $\mathcal{A}$ is any defeat path starting with $\mathcal{A}$ $[\mathcal{A}, \mathcal{D}_1, \mathcal{D}_2, \ldots, \mathcal{D}_n]$. With $DP(\mathcal{A})$ we will denote the set of all defeat paths for $\mathcal{A}$.*

If the length of a defeat path for argument $\mathcal{A}$ is odd, then the last argument in the sequence is playing a *supporting* or *defender* role. If the length is even, then the last argument is playing an *interfering* or *attacker* role [10, 2].

Definition 6 (Supporting and interfering paths). *Let Φ be an argumentation framework, $\mathcal{A}$ an argument in Φ and λ a defeat path for $\mathcal{A}$. If $|\lambda|$ is odd then λ is said to be a* supporting defeat path *for $\mathcal{A}$. If $|\lambda|$ is even, then λ is said to be an* interfering defeat path *for $\mathcal{A}$.*

The notion of defeat path is very simple and only requires that any argument in the sequence must defeat the previous one. Under this unique constraint, which is the basis of argumentation processes, it is possible to obtain some controversial structures, as shown in the next examples.

Example 3. Let $\Phi = \langle AR, \sqsubseteq, \mathbf{C}, \pi \rangle$ an argumentation framework where

$AR = \{\mathcal{A}, \mathcal{B}, \mathcal{C}\}$,
$\mathbf{C}$=$\{\{\mathcal{A}, \mathcal{B}\}, \{\mathcal{B}, \mathcal{C}\}, \{\mathcal{A}, \mathcal{C}\}\}$ and
$\pi(\mathcal{A}, \mathcal{B}) = \{\mathcal{B}\}, \pi(\mathcal{B}, \mathcal{C}) = \{\mathcal{C}\}, \pi(\mathcal{A}, \mathcal{C}) = \{\}$

The sequence $\lambda = [\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{A}]$ is a defeat path in Φ, because $\mathcal{B}$ is a proper defeater of $\mathcal{A}$, $\mathcal{C}$ is a proper defeater of $\mathcal{B}$ and $\mathcal{A}$ and $\mathcal{C}$ are blocking defeaters of each other. The argument $\mathcal{A}$ appears twice in the sequence, as the first and last argument. Note that in order to analyze the acceptance of $\mathcal{A}$, it is necessary to analyze the acceptance of every argument in λ, including $\mathcal{A}$. This is a circular defeat path for $\mathcal{A}$.

Example 4. Let $\Phi = \langle AR, \sqsubseteq, \mathbf{C}, \pi \rangle$ an argumentation framework where

$AR = \{\mathcal{A}, \mathcal{B}, \mathcal{C}\, \mathcal{A}_1{}^-\}$
$\mathbf{C}$=$\{\{\mathcal{A}_1{}^-, \mathcal{B}\}, \{\mathcal{B}, \mathcal{C}\}, \{\mathcal{A}_1{}^-, \mathcal{C}\}\}$ and
$\pi(\mathcal{A}, \mathcal{B}) = \{\mathcal{B}\}, \pi(\mathcal{B}, \mathcal{C}) = \{\mathcal{C}\}, \pi(\mathcal{A}_1{}^-, \mathcal{C}) = \{\}, \pi(\mathcal{A}, \mathcal{C}) = \{\}$

In this framework a subargument of $\mathcal{A}$ is included. By Axiom 1 if $(\mathcal{A}_1{}^-, \mathcal{B}) \in \mathbf{C}$ then also $(\mathcal{A}, \mathcal{B}) \in \mathbf{C}$. The same is true for $(\mathcal{A}, \mathcal{C})$, due the inclusion of $(\mathcal{A}_1{}^-, \mathcal{C})$ in $\mathbf{C}$. According to this, the sequence $\lambda = [\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{A}_1{}^-]$ is a defeat path in Φ, because $\mathcal{B}$ is a proper defeater of $\mathcal{A}$, $\mathcal{C}$ is a proper defeater of $\mathcal{B}$ and $\mathcal{A}_1{}^-$ and $\mathcal{C}$ are blocking defeaters of each other. Note that even when no argument is repeated in the sequence, the subargument $\mathcal{A}_1{}^-$ was already taken into account in the argumentation line, as argument $\mathcal{B}$ is its defeater. This sequence may be considered another circular defeat path for $\mathcal{A}$.

Controversial situations are clear in examples 3 and 4. In the next example some piece of information is repeated in the sequence, but this is not a controversial situation.

Example 5. Let $\Phi = \langle AR, \sqsubseteq, \mathbf{C}, \pi \rangle$ an argumentation framework where

$AR = \{\mathcal{A}, \mathcal{B}, \mathcal{C}\, {\mathcal{A}_1}^-, {\mathcal{A}_2}^-\}$
$\mathbf{C}$=$\{\{{\mathcal{A}_1}^-, \mathcal{B}\}, \{\mathcal{B}, \mathcal{C}\}, \{{\mathcal{A}_2}^-, \mathcal{C}\} \ldots\}$ and
$\pi(\mathcal{A}, \mathcal{B}) = \{\mathcal{B}\}, \pi(\mathcal{B}, \mathcal{C}) = \{\mathcal{C}\}, \pi({\mathcal{A}_2}^-, \mathcal{C}) = \{\}, \pi(\mathcal{A}, \mathcal{C}) = \{\}$

Again, because $({\mathcal{A}_1}^-, \mathcal{B}) \in \mathbf{C}$ then $(\mathcal{A}, \mathcal{B}) \in \mathbf{C}$. Also $(\mathcal{A}, \mathcal{C}) \in \mathbf{C}$, because $({\mathcal{A}_1}^-, \mathcal{B}) \in \mathbf{C}$. According to this, the sequence $\lambda = [\mathcal{A}, \mathcal{B}, \mathcal{C}, {\mathcal{A}_2}^-]$ is a defeat path in Φ, because $\mathcal{B}$ is a proper defeater of $\mathcal{A}$, $\mathcal{C}$ is a proper defeater of $\mathcal{B}$ and ${\mathcal{A}_2}^-$ and $\mathcal{C}$ are blocking defeaters of each other. In this case, a subargument ${\mathcal{A}_2}^-$ of $\mathcal{A}$ appears in the defeat path for $\mathcal{A}$. However, this is not a controversial situation, as ${\mathcal{A}_2}^-$ was not involved in any previous conflict in the sequence. Argument $\mathcal{B}$ is defeating $\mathcal{A}$ just because $({\mathcal{A}_1}^-, \mathcal{B}) \in \mathbf{C}$, and is not related to ${\mathcal{A}_2}^-$. Defeat path λ is correctly structured.

Note that $[\mathcal{A}, \mathcal{C}]$ is also a defeat path for $\mathcal{A}$. In this case, as stated in example 4, ${\mathcal{A}_2}^-$ should not appear in the sequence.

The initial idea of restricting the inclusion of arguments previously considered in the sequence is not enough. The examples 3, 4 and 5 show that the characterization of well-formed argumentation lines requires more restrictions. Two main problematic situations must be taken into account, as shown in figures 4(a) and 4(b). The marked argument is reinserted in the defeat path. In the first case, it appears again as a defeater of $\mathcal{C}$. In the second case, $\mathcal{A}_i$ is indirectly reinserted by including a superargument in the sequence.

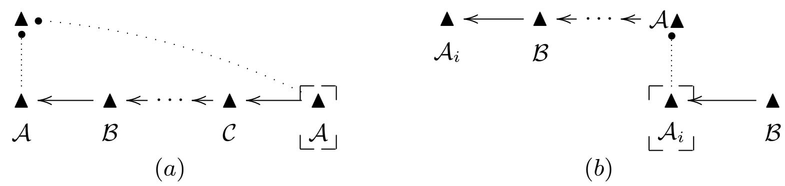

Fig. 4. (a) Direct reinsertion and (b) indirect reinsertion

Both situations are controversial and some well-formed structure must be devised. In the next section we explore these ideas.

4 Progressive Defeat Paths

In this section, we present the concept of progressive defeat paths, a notion related to *acceptable argumentation lines* defined for a particulary concrete system in [2]. This characterization of well-formed defeat path is introduced in the context of our abstract argumentation framework. First, we formalize the consequences of removing an argument from a set of arguments. This is needed because it is important to identify the set of arguments available for use in evolving defeat paths.

Suppose S is a set of available arguments used to construct a defeat path λ. If an argument $\mathcal{A}$ in S is going to be discarded in that process (*i.e.*, its information content is not taken into account), then every argument that includes $\mathcal{A}$ as a subargument should be discarded too.

Definition 7 (Argument extraction). *Let S be a set of arguments and $\mathcal{A}$ an argument in S. The operator $\underline{\triangle}$ is defined as*

$$S \,\underline{\triangle}\, \mathcal{A} = S - Sp(\mathcal{A})$$

where $Sp(\mathcal{A})$ is the set of all superarguments of $\mathcal{A}$.

In figure 5, the extraction of arguments is depicted: $S \,\underline{\triangle}\, \mathcal{A}$ excludes $\mathcal{A}$ and all of its superarguments.

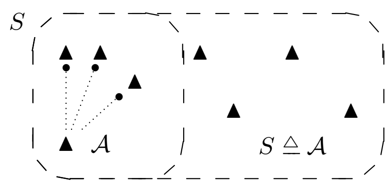

Fig. 5. Argument extraction

Example 6. Let $S = \{\mathcal{A}, \mathcal{A}^+, \mathcal{B}, \mathcal{B}^-, \mathcal{C}\}$ be a set of arguments. Then
$S \,\underline{\triangle}\, \mathcal{A} \;=\; \{\mathcal{B}, \mathcal{B}^-, \mathcal{C}\}$ and
$S \,\underline{\triangle}\, \mathcal{B} \;=\; \{\mathcal{A}, \mathcal{A}^+, \mathcal{B}^-, \mathcal{C}\}$

As stated in Axiom 1, conflict relations are propagated through superarguments: if $\mathcal{A}$ and $\mathcal{B}$ are in conflict, then $\mathcal{A}^+$ and $\mathcal{B}$ are also conflictive arguments. On the other hand, whenever two arguments are in conflict, it is always possible to identify conflictive subarguments. This notion can be extended to defeat relations. Let $\mathcal{A}$ and $\mathcal{B}$ be two arguments such that $\mathcal{B}$ is a defeater of $\mathcal{A}$. Then both arguments are in conflict and $\pi(\mathcal{B}, \mathcal{A}) \neq \{\mathcal{A}\}$. By axiom 1, there may exist a non-trivial subargument $\mathcal{A}_i \sqsubset \mathcal{A}$ such that $(\mathcal{B}, \mathcal{A}_i) \in \mathbf{C}$. It is clear, as π is monotonic, that $\pi(\mathcal{B}, \mathcal{A}_i) \neq \{\mathcal{A}_i\}$, and therefore $\mathcal{B}$ is also a defeater of $\mathcal{A}_i$. Thus, for any pair of conflictive arguments $(\mathcal{A}, \mathcal{B})$ there is always a pair of conflictive arguments $(\mathcal{C}, \mathcal{D})$ where $\mathcal{C} \sqsubseteq \mathcal{A}$ and $\mathcal{D} \sqsubseteq \mathcal{B}$. Note that possibly $\mathcal{C}$ or $\mathcal{D}$ are trivial subarguments, that is the reason for the existence of the pair to be assured.

Definition 8 (Core conflict). *Let $\mathcal{A}$ and $\mathcal{B}$ be two arguments such that $\mathcal{B}$ is a defeater of $\mathcal{A}$. A* core conflict *of $\mathcal{A}$ and $\mathcal{B}$ is a pair of arguments $(\mathcal{A}_i, \mathcal{B})$ where*

- *$\mathcal{A}_i \sqsubseteq \mathcal{A}$,*
- *$\mathcal{B}$ is a defeater of $\mathcal{A}_i$ and*
- *there is no other argument $\mathcal{A}_j \sqsubset \mathcal{A}_i$ such that $\mathcal{A}_j$ is defeated by $\mathcal{B}$.*

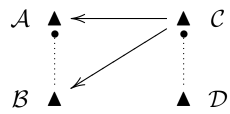

Fig. 6. Argument $\mathcal{B}$ is a core conflict

The core conflict is the underlying cause of a conflict relation between two arguments, due to the inheritance property. Observe that the core conflict is not necessarily unique. It is possible to identify the real disputed subargument, which is causing other arguments to fall in conflict.

In figure 6, argument $\mathcal{C}$ defeats $\mathcal{A}$ because it is defeating one of its subarguments $\mathcal{B}$. The core conflict of $\mathcal{A}$ and $\mathcal{C}$ is $\mathcal{B}$. In this case the defeat arc between the superarguments may not be drawn.

Definition 9 (Disputed subargument). *Let $\mathcal{A}$ and $\mathcal{B}$ be two arguments such that $\mathcal{B}$ is a defeater of $\mathcal{A}$. A subargument $\mathcal{A}_i \sqsubseteq \mathcal{A}$ is said to be a* disputed subargument *of $\mathcal{A}$ with respect to $\mathcal{B}$ if $\mathcal{A}_i$ is a core conflict of $\mathcal{A}$ and $\mathcal{B}$.*

The notion of *disputed subargument* is very important in the construction of defeat paths in dialectical processes. Suppose argument $\mathcal{B}$ is a defeater of argument $\mathcal{A}$. It is possible to construct a defeat path $\lambda = [\mathcal{A}, \mathcal{B}]$. If there is a defeater of $\mathcal{B}$, say $\mathcal{C}$, then $[\mathcal{A}, \mathcal{B}, \mathcal{C}]$ is also a defeat path. However, $\mathcal{C}$ should not be a disputed argument of $\mathcal{A}$ with respect to $\mathcal{B}$, as circularity is introduced in the path. Even more, $\mathcal{C}$ should not be an argument that *includes* that disputed argument, because that path can always be extended by adding $\mathcal{B}$ again.

The set of arguments available to be used in the construction of a defeat path is formalized in the following definition.

Definition 10 (Defeat domain). *Let $\Phi = \langle AR, \sqsubseteq, \mathbf{C}, \pi \rangle$ be an argumentation framework and let $\lambda = [\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n]$ be a defeat path in Φ. The function $D^i(\lambda)$ is defined as*

- $D^1(\lambda) = AR$
- $D^k(\lambda) = D^{k-1}(\lambda) \triangleq \mathcal{B}_n$*, where $\mathcal{B}_n$ is the disputed subargument of $\mathcal{A}_{k-1}$ with respect to $\mathcal{A}_k$ in the sequence, with $2 \leq k \leq n$.*

The defeat domain discards controversial arguments. The function $D^k(\lambda)$ denotes the set of arguments that can be used to extend the defeat path λ at stage k, *i.e.*, to defeat the argument $\mathcal{A}_k$. Choosing an argument from $D^k(\lambda)$ avoids the introduction of previous disputed arguments in the sequence. It is important to remark that if an argument including a previous disputed subargument is reintroduced in the defeat path, it is always possible to reintroduce its original defeater.

Therefore, in order to avoid controversial situations, any argument $\mathcal{A}_i$ of a defeat path λ should be in $D^{i-1}(\lambda)$. Selecting an argument outside this set implies the repetition of previously disputed information. The following definition characterizes well structured sequences of arguments, called *progressive defeat paths*.

Definition 11 (Progressive defeat path). *Let $\Phi = \langle AR, \sqsubseteq, \mathbf{C}, \pi \rangle$ be an argumentation framework. A progressive defeat path is defined recursively in the following way:*

- *$[\mathcal{A}]$ is a progressive defeat path, for any $\mathcal{A} \in AR$.*
- *If $\lambda = [\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n]$, $n \geq 1$ is a progressive defeat path, then for any defeater $\mathcal{B}$ of $\mathcal{A}_n$ such that $\mathcal{B} \in D^n(\lambda)$, $\lambda' = [\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n, \mathcal{B}]$ is a progressive defeat path.*

Observe that defeat paths of examples 3 and 4 are not progressive. Progressive defeat paths are free of circular situations and guarantees progressive argumentation, as desired on every dialectical process. Note that it is possible to include a subargument of previous arguments in the sequence, as long as it is not a disputed subargument.

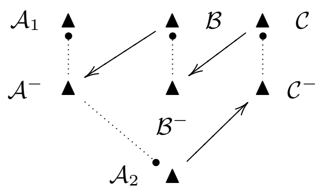

Fig. 7. Controversial Situation

In figure 7 a controversial abstract framework is shown. For space reasons we do not provide the formal specification, although it can be deduced from the graph. There are seven arguments $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}^-, \mathcal{B}, \mathcal{B}^-, \mathcal{C}, \mathcal{C}^-$. There exists an infinite defeat path $[\mathcal{A}_1, \mathcal{B}, \mathcal{C}, \mathcal{A}_2, \mathcal{B}, \mathcal{C}..]$ which is not progressive. Lets construct a progressive defeat path λ for argument $\mathcal{A}_1$. We start with $\lambda = [\mathcal{A}_1]$. The pool of arguments used to select a defeater of $\mathcal{A}_1$ is $D^1(\lambda) = \{\mathcal{A}_2, \mathcal{A}^-, \mathcal{B}, \mathcal{B}^-, \mathcal{C}, \mathcal{C}^-\}$. The only defeater belonging to $D^1(\lambda)$ is $\mathcal{B}$, with disputed subargument $\mathcal{A}^-$, so we add it to λ. Now $\lambda = [\mathcal{A}_1, \mathcal{B}]$ and the pool of available arguments is $D^2(\lambda) = \{\mathcal{B}, \mathcal{B}^-, \mathcal{C}, \mathcal{C}^-\}$, where $\mathcal{A}^-$ and its superarguments were removed. $\mathcal{C} \in D^2(\lambda)$ is a defeater of $\mathcal{B}$ so we add it to the path and now $\lambda = [\mathcal{A}_1, \mathcal{B}, \mathcal{C}]$. The potential defeater arguments are now in $D^3(\lambda) = \{\mathcal{C}, \mathcal{C}^-\}$. As there are no defeaters of $\mathcal{C}$ in $D^3(\lambda)$, then the path can not be extended. Thus, the resulting sequence $[\mathcal{A}_1, \mathcal{B}, \mathcal{C}]$ is a progressive defeat path.

5 Conclusions

Abstract argumentation systems are formalisms for argumentation, where some components remains unspecified, usually the structure of arguments. In the dialectical process carried out to identify accepted arguments in the system, some controversial situations may be found, related to the reintroduction of arguments in this process, causing a circularity that must be treated in order to avoid an infinite analysis process. Some systems apply a single restriction to argumentation lines: no previously considered argument is reintroduced in the process. In this work, we have shown that a more specific restriction need to be applied, taking subarguments into account in the context of an extended argumentation framework. We finally presented a new definition of *progressive defeat path*, based on the concept of *defeat domain*, where superarguments of previously disputed arguments are discarded.

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